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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

Fedor Ivanovitsch Blumbach, Chief Metrologist of the Central Chamber of Weights and Measures, Leningrad, Russia;
Mary Acworth Evershed, Highbroom, Ewhurst, near Guildford, Surrey;
Alexei Nikolaievitsch Kriloff, Member of the Russian Academy of Sciences, National Hotel, Upper Bedford Place, London, W.C. 1;
Clayton Conyers Morrell, M.D.Brux., L.R.C.S., L.R.C.P. Edin., D.P.H.Lond., 28 Stratford Place, Camden Square, London, N.W. 1; and
William R. White, City Engineer's Office, Bloemfontein, O.F.S., S. Africa,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

G. van Biesbroeck, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A. (proposed by F. W. Dyson);
Frank Hope-Jones, M.I.E.E., 124 Queen's Road, Richmond, Surrey (proposed by F. W. Dyson); and
Kennet Pollock, 48A Stondon Park, Forest Hill, London, S.E. 23 (proposed by E. W. Maunder).

Forty-nine presents were announced as having been received since the last meeting.

The Declinations of the Greenwich Observations for 1915–21.

By Sir Frank Dyson and Mr. R. T. Cullen.

1. The large corrections to the declinations of Newcomb's Fundamental Catalogue given in the American Ephemeris for 1925, resulting from Professor Eichelberger's discussion of recent Washington and Cape Catalogues, invite further investigation. Roughly speaking, they give a correction of $+''\cdot25$ to the equator point of Newcomb and Auwers and $+''\cdot50$ to that of Boss. On this account it seems desirable to give a brief exposition of the declinations of the forthcoming Greenwich Catalogue as far as they bear on this important question. The stars observed are those in the Backlund-Hough list, and give a very uniform distribution of moderately bright stars. The observations were made in the years 1915–21, and have been reduced to the epoch 1925·0 with Newcomb's precessions and Boss's proper motions, or, for stars not in the P.G.C., Nyren's proper motions (*Kung. Svenska Vets. Hand.*, Band 60, No. 52, Stockholm) revised at Greenwich by the addition of recent Greenwich and Cape observations.

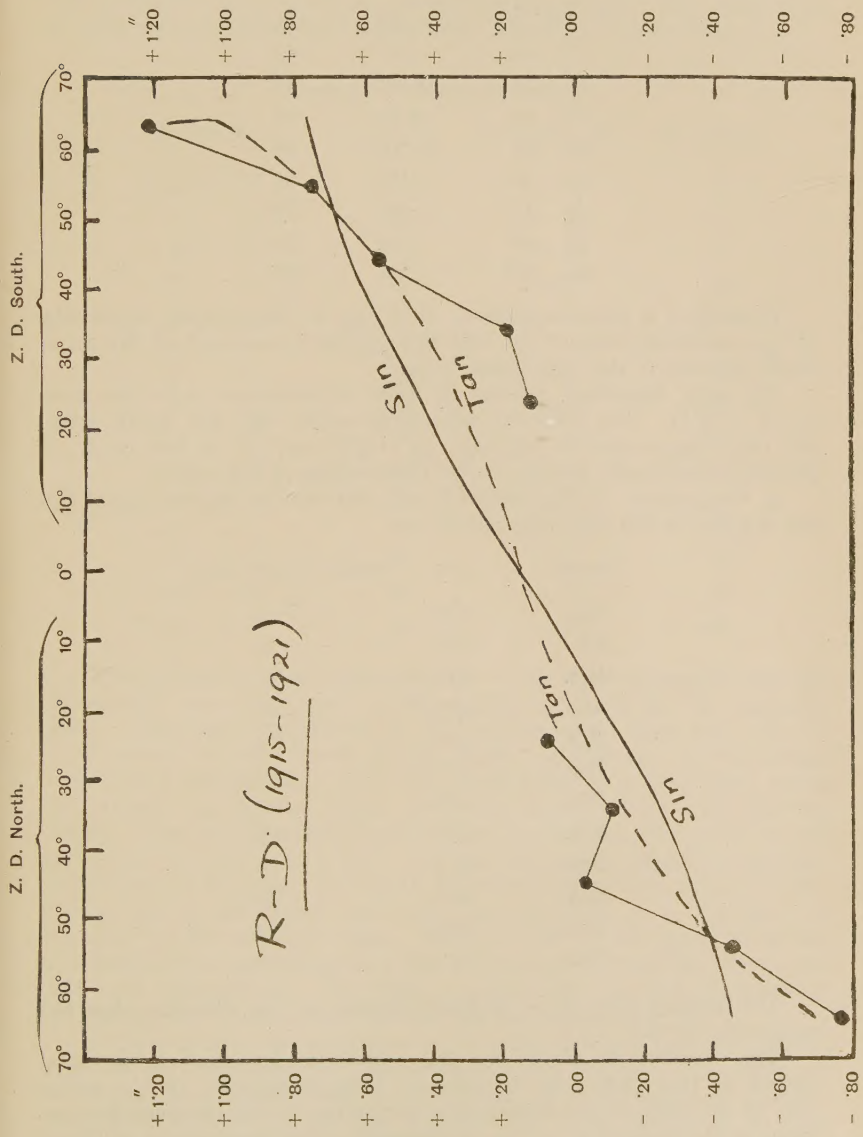
2. The perennial difficulty in the treatment of the Greenwich observations is the R – D discordance. The flexure shown by observations of the collimators is small, but differences are always shown between observations taken by reflexion from mercury and those taken directly. For the years 1915–21 these are found to be as follows:—

North Stars.				South Stars.			
Z.D.	R–D.	Residuals.		Z.D.	R–D.	Residuals.	
		Sin.	Tan.			Sin.	Tan.
$-24^\circ\cdot4$	$+''\cdot08$	$-''\cdot21$	$-''\cdot12$	$+24^\circ\cdot8$	$+''\cdot13$	$+''\cdot31$	$+''\cdot21$
$-34\cdot0$	$-\cdot11$	$-\cdot12$	$-\cdot02$	$+34\cdot5$	$+\cdot19$	$+\cdot35$	$+\cdot25$
$-45\cdot1$	$-\cdot03$	$-\cdot30$	$-\cdot24$	$+45\cdot1$	$+\cdot56$	$+\cdot08$	$+\cdot01$
$-54\cdot9$	$-\cdot46$	$+\cdot06$	$+\cdot02$	$+55\cdot6$	$+\cdot75$	$-\cdot03$	$+\cdot01$
$-63\cdot9$	$-\cdot78$	$+\cdot32$	$+\cdot08$	$+64\cdot0$	$+1\cdot22$	$-\cdot46$	$-\cdot21$

Equating these differences to a formula $a + b \sin z$, we find $a = +''\cdot155$, $b = +''\cdot69$, and the residuals are shown in the column sin. It cannot be said that the observations fit the formula at all well. In fact, a formula $a + b \tan z$, giving $a = +''\cdot152$, $b = +''\cdot418$, fits the observations twice as well. The formula $a + b \sin z$ may be regarded as of the right form if the discordance arises from flexure, and the observations give the coefficients a and b, but they do not verify the form of the law. The diagram given below, though obtained from the observations in 1915–21, is typical of other years.

3. The justification for a correction of the form $a + b \sin z$ to the Greenwich zenith distances is undoubtedly that observations of the sun and planets are thus satisfied. The value of the coefficient of the sin term has varied a good deal in different determinations. Newcomb used $0''\cdot50 \sin z$ and the refractions of Bessel. In the later Greenwich

Catalogues $+0''{\cdot}60 \sin z$ and the Pulkovo refractions have been used, and the present catalogue has been reduced on this basis.



4. Assuming a colatitude 38° 31′ 21″·90, and applying to the observed zenith distances the Pulkovo refractions and a term $+''{\cdot}6 \sin z$, we find for the circumpolars :—

Differences of N.P.D. (Above – Below).

N.P.D.		Above – Below.	No of Stars.
°	°	″	
0 to	5	–·18	21
5	10	+·17	43
10	15	+·06	46
15	20	+·13	47
20	25	+·14	58
25	30	+·19	63
30	35	+·08	88
35	40	–·03	72
40	42½	–·21	52

These give a mean correction of –″·04 to the adopted colatitude 21″·90, excluding stars of the last two groups on account of the large zenith distance of the stars below pole.

The only discordant feature in these observations is in the zone 0° – 5° N.P.D. The Pulkovo refractions satisfy the low north stars, and the disagreement of the zone 40° – 42½°, as will be seen later, is probably attributable to error in the observations at the zenith.

5. Comparison of the observed with the tabular declinations of the sun is given in the following results :—

Month.	Z.D.	Declination (Obs. – Tab.).
	°	″
Jan.	72·5	+·22
Feb.	63·5	+·23
Mar.	52·5	+·10
Apr.	41·5	+·03
May	32·5	+·14
June	28·5	–·05
July	30·5	–·13
Aug.	38·5	–·05
Sept.	48·5	–·28
Oct.	60·5	–·30
Nov.	69·5	–·16
Dec.	74·5	+·10

The mean of these is –″·01, and becomes +″·03 with the observed colatitude 21″·86.

The observations of the sun thus appear to be satisfactorily represented by this system of reductions. With colatitude 21″·90, which will be adopted in the catalogue, a correction to the observed declinations at the equator of +″·01 is indicated. The observations of Venus discussed by Dr. Jackson in the following paper (p. 480) give a correction of +″·18 to the declinations.

6. The *Second Cape Catalogue for* 1900, consisting of observations

made in the years 1912–16, is near the same epoch, and contains over 1200 stars common to both catalogues. The comparison between the two catalogues has, therefore, a considerable weight. The differences of declination in the sense "Greenwich – Cape" are given below :—

Declinations (Greenwich – Cape).

Limits of Dec.		Z.D. at Greenwich.	Greenwich – Cape.	No. of Stars.
°	°	°	″	
+45	to 40	9	−·06	15
40	35	14	−·19	23
35	30	19	+·03	77
30	25	24	+·05	88
25	20	29	·00	89
20	15	34	+·07	84
15	10	39	+·21	91
10	5	44	+·16	102
+ 5	0	49	+·28	91
− 0	− 5	54	+·31	95
− 5	−10	59	+·23	96
−10	−15	64	+·35	90
−15	−20	69	+·45	96
−20	−25	74	+·42	91
−25	−30	79	+·70	102

The gradient of these differences below the equator suggests that the Greenwich refractions need to be slightly increased for the southern stars. But the observations of circumpolar stars show that this is inadmissible for stars north of the zenith. A difference in the refractions seems very probable, as to the south there is a level plain, while to the north the ground slopes down very rapidly to the river 150 feet below. The meridian reaches the river 1100 yards away, and follows its course for a mile and a half. It seems reasonable, therefore, that the refractions on the south should be somewhat larger than those on the north, and an additional correction for refraction of +″·10 tan z is required by the southern stars. This gets rid of the gradient at the low declinations and straightens the differences between the Greenwich and Cape observations. The effect of this change on the observations of the sun will in the mean diminish the declination by 0″·16.

Adding this to the correction +″·01 already found, with this alteration of the refraction, the declinations would require a correction of +″·17 in the neighbourhood of the equator. The difference of declination (G – C) is +″·10 at the equator with this correction to the Greenwich refractions. Also in the *Cape Annals*, 8, pt. 5, p. 29E, it is shown that the observations of the Sun, Mercury, and Venus at the Cape indicate a considerable positive correction to the Cape declinations.

7. Taking these somewhat discordant facts into consideration, the

best general correction to the Greenwich Catalogue is a small diminution of the coefficient of sin z, and an additional correction of $+''\cdot10 \tan z$ to the south stars only. Adopting $21''\cdot80$ for the colatitude, $+''\cdot50 \sin z$, Pulkovo refractions, with an additional $+''\cdot10 \tan z$ for the south stars only, we find for the circumpolars:—

N.P.D.	N.P.D. (Above – Below)	
° °	″	
0– 5	–·25	Correction to colat. $21''\cdot80$ is $''\cdot00$.
5–10	+·09	
10–15	–·02	
15–20	+·05	
20–25	+·06	
25–30	+·10	
30–35	–·01	
35–40	–·13	
40–42·5	–·32	

Observations of the Sun.

Month.	Dec. (Obs. – Tab.).	Month.	Dec. (Obs. – Tab.).
	″		″
Jan.	+·10	July	–·04
Feb.	+·22	Aug.	+·03
Mar.	+·15	Sept.	–·22
Apr.	+·11	Oct.	–·29
May	+·23	Nov.	–·24
June	+·05	Dec.	–·06

The mean of these quantities is zero. A comparison was made of the observations of stars by day and night. The difference was found to be $+''\cdot04 \pm ''\cdot08$. No correction was considered necessary on this account.

Comparison with Cape Declinations.

Limits of Dec.	G – C.	Limits of Dec.	G – C.	Limits of Dec.	G – C.
° to °	″	° to °	″	° to °	″
45 to 40	+·04	20 to 15	+·16	– 5 to – 10	+·25
40 35	–·10	15 10	+·29	–10 –15	+·34
35 30	+·12	10 5	+·23	–15 –20	+·37
30 25	+·14	5 0	+·34	–20 –25	+·26
25 20	+·09	–0 –5	+·35	–25 –30	+·36

The mean of the differences G – C from $+20°$ to $-30°$ is $+''\cdot30$. For the years 1911–16 Hough found from very accordant observations of the Sun, Venus, and Mercury a correction of $+''\cdot47$ to the Cape declinations in the neighbourhood of the equator. For the years 1905–11 a correction of $+''\cdot25$ was obtained, and allowing for the systematic difference in the declinations of the two catalogues, becomes

+0″·42. This will give G − C = −″·17 from 1912–16 and −″·12 from 1905–11.

On the assumption that the most satisfactory way of reducing declinations is to utilise observations of the sun and planets to fix the equator-point, it would seem that 21″·90 + ″·6 sin z + Pulkovo refractions suits the Greenwich observations well, both stars and sun, except for the declinations −10° to −30°. This system, which will be adopted in the printed catalogue, is improved by alteration to 21″·80 + ″·5 sin z + Pulkovo refractions + an additional refraction term + ″·10 tan z for the stars south of the zenith. The numerical amount of these corrections is shown in the following table:—

Dec.	Corr.	Dec.	Corr
+50°	+″·10	+60°	+″·09
40	+·10	70	+·07
30	+·10	80	+·05
20	+·09	90	+·04
10	+·08	S.P.	
0	+·05	80	−·03
−10	+·01	70	−·02
−20	−·11	60	−·01
−30	−·47	50	·00

8. With these corrections applied to the Greenwich declinations, differences are formed from the 1915 Pulkovo Catalogue (1911–16), by Bonsdorff; from the Fundamental Catalogues of Auwers, as obtained by comparison with the Berliner Jahrbuch for 1925; the Preliminary General Catalogue of Boss; and Newcomb's Catalogue with the corrections given in the American Ephemeris for 1925. They show that the Greenwich declinations are in general agreement with the Berliner Jahrbuch in taking a middle position between those of Boss and the American Ephemeris of 1925, pp. 750–764.

Declinations (Greenwich − Catalogue).

Dec.		B.J.	Boss.	Pulkovo.	A.E.
+90° to	80°	−″·10	″·00	+″·00	″
80	70	−·08	+·01	−·03	−·03
70	60	−·11	−·06	−·04	−·13
60	50	−·09	+·06	+·02	−·26
50	40	+·34	+·41	+·28	+·04
40	30	−·10	+·37	+·25	−·15
30	20	−·01	+·29	+·18	−·24
20	10	+·07	+·28	+·23	−·29
10	0	+·05	+·18	+·28	−·35
−0	−10	−·01	+·12	+·15	−·24
−10	−20	+·03	+·25	...	−·16
−20	−30	+·18	+·30	...	−·23

9. The discordance in zone 50° to 40° requires notice. This is close to the Greenwich zenith. The differences for zones 3° wide near the Greenwich zenith are given in detail in the following table:—

Limits of Dec.	Greenwich – Pulkovo.	Greenwich – Boss.	Greenwich – Auwers.
+59° to 56°	−″·10	−″·07	−″·14
56 53	−·06	−·07	−·21
53 50	−·01	+·17	+·02
50 47	+·08	+·30	+·21
47 44	+·18	+·33	+·29

Bisection error of the observers was suspected as a likely cause, and observations were made with a reversing eye-piece. These, however, gave a negative result. The rapid change is shown by the recent Pulkovo and Washington observations, as well as by the Fundamental Catalogues.

10. It may be pointed out that the value of $\Delta\delta_0$ shown by the sun and planets at the Cape could be corrected by the application of an empirical term of approximately $''\cdot 3 \sin z$, and this would bring agreement with Greenwich near the equator. But the low north stars at the Cape would then differ more from the high stars at Greenwich and a correction for refraction would be required. A correction $+''\cdot 40 \sin z - ''\cdot 10 \tan z$ to the Cape Catalogue of 1912–16 would give a fair accordance.

A Determination of Corrections to the Equinox and Equator Point from Observations of Venus made at the Royal Observatory, Greenwich, 1915–1923. By J. Jackson, M.A., D.Sc.

(*Communicated by the Astronomer Royal.*)

As the first point of Aries is defined by the intersection of the equator and the ecliptic, it has to be determined from observations which reflect the motion of the earth round the sun. This has generally been done from observations of the sun, although Newcomb * made extensive use of observations of the inner planets for this purpose. The principle underlying the use of these observations is that an observation of a neighbouring planet is equivalent to an observation of the earth from that planet. The equations of condition involve the elements of both orbits. Although the normal equations do not separate well the eccentricity and perihelion of the earth's orbit from the same elements of the planet's orbit, it is found that they give a good determination of the equinox and of the earth's longitude. In this connection Newcomb wrote: "Systematic and personal errors enter so largely that the results from Mercury and Venus appear to be rather more reliable than those from the Sun itself." Recently Hough † has discussed the observations

* "The Elements of the Four Inner Planets."

† *Cape Annals*, 8, parts 4 and 5.

of the Sun, Mercury and Venus made at the Cape Observatory in the years 1907–1916, and in this paper a similar plan is followed in discussing the observations of Venus made in the years 1915–1923 with the Greenwich Transit Circle. The observations of the Sun have been discussed for the determination of the equinox in the Greenwich Volumes up to 1914, and since then the results have been collected and are given in the previous paper (p. 472). In England it is difficult to secure a regular series of observations of Mercury, as in winter the planet can only be observed on rare occasions.

The right ascensions depend on the adopted positions of the clock stars, the declinations were reduced with a latitude of 51° 28′ 38″·10, Pulkowa refractions, a term 0″·60 sine zenith distance supposed to represent flexure, and the latitude variation determined with the Cookson Floating Telescope.

There were in all 438 observations of Venus in the nine years. The residuals from the N.A. positions were first of all converted into longitude and latitude. The residuals in this paper, $\Delta\lambda$, $\Delta\beta$, are given in the sense Observed *minus* Computed. The analysis of the residuals was made by sorting them into groups in three different ways according to the values of l, L, and $l - L$, where l, L are the heliocentric longitudes of Venus and the earth. The following tables give the mean value of $\Delta\beta$ and the number of observations for each group.

l.	$\Delta\beta$.	No. of Obs.	l.	$\Delta\beta$.	No. of Obs.
° °	″		° °	″	
0–30	− ·06	38	180–210	+ ·01	37
30–60	− ·25	39	210–240	− ·10	24
60–90	− ·18	37	240–270	+ ·08	29
90–120	− ·02	36	270–300	+ ·07	37
120–150	− ·33	50	300–330	− ·14	43
150–180	+ ·15	33	330–360	+ ·24	35

L.	$\Delta\beta$.	No. of Obs.	L.	$\Delta\beta$.	No. of Obs.
° °	″		° °	″	
0–30	+ ·38	50	180–210	− ·32	25
30–60	− ·34	26	210–240	− ·29	44
60–90	− ·74	17	240–270	− ·03	73
90–120	− ·42	19	270–300	− ·01	39
120–150	− ·77	38	300–330	+ ·48	52
150–180	− ·22	19	330–360	+ ·46	36

l − L.	$\Delta\beta$.	No. of Obs.	l − L.	$\Delta\beta$.	No. of Obs.
° °	″		° °	″	
0–30	+ ·24	38	180–210	− ·12	16
30–60	− ·38	30	210–240	+ ·28	28
60–90	− ·25	60	240–270	− ·26	36
90–120	− ·12	55	270–300	+ ·33	36
120–150	− ·38	33	300–330	− ·21	36
150–180	+ ·14	28	330–360	+ ·15	42

As most of the Greenwich observations in declination were made at both N. and S. limbs, the above results should be nearly independent of the accepted semi-diameter. It is doubtful if the adopted correction for cusps was always strictly applicable. The influence of errors of this kind would, however, probably be small, and be involved in the coefficient of sin $(l - \mathrm{L})$ in the analysis. When the values of $\Delta\beta$ given in the above tables were analysed they gave the following results, where the probable errors were obtained from the residuals, equal weight being given to each mean.

$$\begin{aligned}
(1)\ \Delta\beta = &-''\cdot04 + ''\cdot00\cos l && -''\cdot10\sin l\\
&\pm\cdot03 \pm\cdot04 && \pm\cdot04.\\
(2)\ \Delta\beta = &-\cdot15 + \cdot35\cos \mathrm{L} && -\cdot33\sin \mathrm{L}\\
&\pm\cdot05 \pm\cdot07 && \pm\cdot07.\\
(3)\ \Delta\beta = &-\cdot05 + \cdot02\cos(l-\mathrm{L}) && -\cdot15\sin(l-\mathrm{L})\\
&\pm\cdot05 \pm\cdot07 && \pm\cdot07
\end{aligned}$$

Hence we may take

$$\begin{aligned}
\Delta\beta = &-''\cdot08 + ''\cdot00\cos l && -''\cdot10\sin l\\
&+\cdot35\cos \mathrm{L} && -\cdot33\sin \mathrm{L}\\
&+\cdot02\cos(l-\mathrm{L}) && -\cdot15\sin(l-\mathrm{L}).
\end{aligned}$$

The longitudes, when treated in the same way, gave the following mean residuals :—

l.	$\Delta\lambda$.	No. of Obs.	l.	$\Delta\lambda$.	No. of Obs.
0°–30°	+1″·49	38	180°–210°	+1″·04	37
30–60	+1·67	39	210–240	+1·07	24
60–90	+1·95	37	240–270	+1·28	29
90–120	+2·06	36	270–300	+0·95	37
120–150	+1·16	50	300–330	+1·44	43
150–180	+1·24	33	330–360	+0·89	35

L.	$\Delta\lambda$.	No. of Obs.	L.	$\Delta\lambda$.	No. of Obs.
0°–30°	+1″·61	50	180°–210°	+0″·89	25
30–60	+1·74	26	210–240	+1·73	44
60–90	+1·08	17	240–270	+1·50	73
90–120	+0·99	19	270–300	+2·00	39
120–150	+0·57	38	300–330	+1·35	52
150–180	+0·89	19	330–360	+1·10	36

l–L.	$\Delta\lambda$.	No. of Obs.	l—L.	$\Delta\lambda$.	No. of Obs.
0°–30°	+0″·92	38	180°–210°	+2″·22	16
30–60	+2·22	30	210–240	+1·32	28
60–90	+2·21	60	240–270	+1·38	36
90–120	+2·22	55	270–300	+0·97	36
120–150	+2·21	33	300–330	+0·26	36
150–180	+2·20	28	330–360	−1·46	42

The analysis of these values gave the following:—

$$\begin{aligned}
(1)\ \Delta\lambda = &+ 1''\cdot35 + ''\cdot10\cos l && + ''\cdot36\sin l\\
&\pm\ \cdot05 \pm \cdot08 && \pm \cdot08.\\
(2)\ \Delta\lambda = &+ 1\cdot29 + \cdot27\cos \mathrm{L} && - \cdot30\sin \mathrm{L}\\
&\pm\ \cdot06 \pm \cdot08 && \pm \cdot08.\\
(3)\ \Delta\lambda = &+ 1\cdot39 - \cdot94\cos(l-\mathrm{L}) && + \cdot78\sin(l-\mathrm{L})\\
&\pm\ \cdot11 \pm \cdot16 && \pm \cdot16.
\end{aligned}$$

From which we may take

$$\begin{aligned}
\Delta\lambda = &+ 1''\cdot34 + ''\cdot10\cos l && + ''\cdot36\sin l\\
&+ \cdot27\cos \mathrm{L} && - \cdot30\sin \mathrm{L}\\
&- \cdot94\cos(l-\mathrm{L}) && + \cdot78\sin(l-\mathrm{L}).
\end{aligned}$$

Hough has shown that for Venus $\Delta\beta, \Delta\lambda$ take the following forms:—

$$\begin{aligned}
\Delta\beta = \Delta\delta_0 &+ \left[0\cdot855\Delta\alpha_0\sin\omega - 0\cdot348\Delta q\right]\cos \mathrm{L} + \left[0\cdot855\Delta\omega + 0\cdot348\Delta p\right]\sin \mathrm{L}\\
&- \left[0\cdot389\Delta\alpha_0\sin\omega + 0\cdot870\Delta q\right]\cos l - \left[0\cdot389\Delta\omega - 0\cdot870\Delta p\right]\sin l.
\end{aligned}$$

$$\begin{aligned}
\Delta\lambda = \Delta\alpha_0 &+ \Delta\mathrm{E} + 0\cdot723(\Delta\mathrm{E} - \Delta\epsilon)\cos(l-\mathrm{L})\\
&- \left[0\cdot362\Delta\mathrm{K} - 0\cdot389\Delta\delta_0\right]\cos l + 0\cdot362\Delta\mathrm{H}\sin l\\
&- \left[2\Delta\mathrm{K} - 1\cdot086\Delta k + 0\cdot340\Delta\delta_0\right]\cos \mathrm{L}\\
&+ 2\left[\Delta\mathrm{H} - 1\cdot086\,\Delta\mathrm{L}\right]\sin \mathrm{L},
\end{aligned}$$

where $-\Delta\alpha_0$, $-\Delta\delta_0$ are corrections which should be applied to our adopted right ascensions and to our system of declinations; $\Delta\mathrm{E}$, $\Delta\epsilon$ are corrections of the mean longitudes of the earth and Venus, and the other quantities are in the usual notation for corrections to the orbit elements.

We thus find

$$\begin{aligned}
0\cdot855(\Delta\alpha_0\sin\omega) - 0\cdot348\Delta q &= + 0''\cdot35 \pm ''\cdot07\\
0\cdot389(\Delta\alpha_0\sin\omega) + 0\cdot870\Delta q &= + 0\cdot00 \pm \cdot04\\
\therefore\ \Delta\alpha_0 &= + 0\cdot87 \pm \cdot18 = + {}^{s}\cdot058 \pm {}^{s}\cdot012.
\end{aligned}$$

Also

$$\Delta\alpha_0 + \Delta\mathrm{E} = + 1''\cdot34.$$

Adopting $\Delta\alpha_0 = + 0''\cdot87$ we find $\Delta\mathrm{E} = + 0''47$.

We thus find that the adopted right ascensions are too large by $+ 0^{s}\cdot058$, the declinations are too small by $+ 0''\cdot08$, and that the longitude of the sun should be increased by $+ 0''\cdot47$. It might be mentioned here that any error in the declinations of the form $a \times \cos$ R.A. will introduce an error in the determination of the equinox. The correction to the obliquity indicated by these observations is $- 0''\cdot29 \pm ''\cdot07$.

The declinations may be treated directly without changing into longitude and latitude. When grouped according to R.A. and L they give the following means, based on 472 observations.

R.A.	Δδ.	No. of Obs.	R.A.	Δδ.	No. of Obs.
h h	″		h h	″	
0-2	−·19	38	12-14	+·84	24
2-4	+·12	49	14-16	+·04	25
4-6	+·06	49	16-18	−·23	24
6-8	−·02	65	18-20	−·53	23
8-10	−·34	42	20-22	−·31	36
10-12	+·24	70	22-24	+·02	28

L.	Δδ.	No. of Obs.	L.	Δδ.	No. of Obs.
° °	″		° °	″	
0-30	−·26	54	180-210	+·02	30
30-60	−·85	28	210-240	+·17	46
60-90	−·90	18	240-270	+·18	76
90-120	−·48	20	270-300	+·07	43
120-150	−·74	40	300-330	+·22	57
150-180	+·04	21	330-360	+·28	39

These values give on analysis

$$\Delta\delta = -0''.16 + ''.07 \cos \text{R.A.} + ''.17 \sin \text{R.A.}$$
$$\pm\ .05 \pm .07 \qquad \pm .07$$
$$\Delta\delta = -0''.19 - .04 \cos \text{L} \quad - .52 \sin \text{L}$$
$$\pm\ .05 \pm .07 \qquad \pm .07.$$

This indicates that the Greenwich declinations should be increased by $+0''.18$ as compared with $+0''.08$ deduced from the discussion of the latitudes.

Observations of Mercury.

When the residuals in declination for Mercury were grouped according to R.A. and L they gave the following results :—

R.A.	Δδ.	No. of Obs.	R.A.	Δδ.	No. of Obs.
h h	″		h h	″	
0-2	+·47	16	12-14	−·92	23
2-4	+·69	18	14-16	...	0
4-6	+·03	12	16-18	−1·02	2
6-8	−·02	13	18-20	·08	1
8-10	−·25	8	20-22	−·64	2
10-12	−·01	20	22-24	...	0

L.	Δδ.	No. of Obs.	L.	Δδ.	No. of Obs.
° °	″		° °	″	
0-30	−·88	17	180-210	+·16	10
30-60	−1·11	7	210-240	+·79	17
60-90	...	0	240-270	+·28	21
90-120	−1·02	2	270-300	+·08	9
120-150	...	0	300-330	+·05	14
150-180	−·32	4	330-360	−·21	14

It is hardly worth while analysing these observations, as there are very few between 14^h and 24^h of R.A. or between 30° and 180° of L. The simple mean for $\Delta\delta$ is $-''\cdot06$, indicating a correction of $+0''\cdot06$ at the equator as compared with $+''\cdot18$ from the observations of Venus.

The Systematic Motions of Faint Stars. Third Paper.

By W. M. Smart, D.Sc.

(*Communicated by the Director of Cambridge University.*)

§ 1. In two previous papers (*M.N.*, **84**, 3; **84**, 123) I have investigated the systematic motions of faint stars in 8 photographic regions. The general conclusion based on the results for these regions is that the phenomenon of star-streaming is a common feature of the fainter and of the brighter stars. Similar results have been obtained for the southern stars by Dr. Innes at the Union Observatory (*M.N.*, **84**, 190). It is true that there are apparent anomalies, the most conspicuous of which has been found in the distribution of the motions of the stars in the region around ρ Persei (*M.N.*, **84**, 125). In this paper three further regions are examined; the second—that of β Persei—is contiguous to the region of ρ Persei, and it is a matter of considerable interest to compare the distributions of motions in these neighbouring regions. The regions are as follows:—

Number.	Region.	R.A. (1900)	Dec.	No. of Stars.	Interval.
		h m	° ′		
I.	Groombridge 34	0 12·7	+43 27·4	174	18·6 years
II.	β Persei	3 1·7	+40 34·2	78	18·3 ,,
III.	Lalande 46650	23 44·0	+ 1 52·3	42	18·5 ,,

The name of the central star with its approximate co-ordinates for 1900·0 is given in each case. In the last column are the intervals between the early plates (direct) and the recent plates (photographed through the glass), and in the penultimate column are the numbers of stars for which proper motions have been determined. The measures have been made independently by Mr. H. E. Green and the writer. The third region is in a high galactic latitude, and the number of stars is hardly adequate to give more than an approximate idea of the general systematic motions in this part of the sky.

The method of reduction and the analysis of the proper motions are essentially those of the previous papers. For 2 regions I give below diagrams to show the measured relative proper motions. The absolute motions have been derived mainly by comparing the parallactic motions of the B.D. stars in the separate fields with the relative proper motions. There are very few stars in the regions considered with well-determined meridian proper motions, and accordingly the values of ξ and η—the

co-ordinates of the origin A of absolute motions with reference to the origin R of relative motions—are almost completely dependent on parallactic considerations. In the diagrams referred to above the origins A and R are shown.

The following table gives the number of B.D. stars in each region which were used in deriving the values of ξ and η by comparing the mean relative proper motions with the mean parallactic motions:—

TABLE I.

Region.	No. of B.D. Stars.
I.	44
II.	35
III.	25

§ 2. *Distribution of the Proper Motions in Direction.*—Table II. gives the number of stars moving in each of the 36 sectors $\theta - 5^\circ$ to $\theta + 5^\circ$ ($\theta = 10^\circ, 20^\circ, \ldots 360^\circ$). The column II*a* gives the total number of stars, moving in each of the sectors, for the region of β Persei combined with that of ρ Persei (*M.N.*, **84**, 124; Table II.). The curves $r = f(\theta)$ given below have as radius vector, r, the number of stars in the 30° sectors $\theta - 15^\circ$ to $\theta + 15^\circ$ ($\theta = 10^\circ, 20^\circ, \ldots 360^\circ$).

TABLE II.

Distribution of Proper Motions in Direction.

	Region.					Region.			
θ.	I.	II.	II*a*.	III.	θ.	I.	II.	II*a*.	III.
10°	3	0	1	0	190°	13	0	4	0
20	2	1	1	0	200	3	7	7	1
30	2	0	0	0	210	6	5	7	2
40	0	0	1	1	220	2	2	3	1
50	4	0	0	2	230	3	2	3	2
60	5	2	5	3	240	0	0	2	1
70	8	0	1	2	250	1	1	1	0
80	9	1	4	4	260	0	2	2	0
90	10	2	5	4	270	0	1	2	0
100	22	4	8	1	280	1	1	2	0
110	15	5	8	2	290	0	1	1	0
120	12	7	11	3	300	1	1	2	0
130	10	4	11	2	310	0	0	1	0
140	13	5	11	1	320	0	0	0	1
150	11	10	14	0	330	0	1	1	1
160	7	2	6	2	340	0	0	0	0
170	6	5	9	2	350	0	1	1	2
180	3	5	6	2	360	2	0	2	0

§ 3. In the following paragraphs I give for each region the galactic latitude, position angles of the solar ant-apex and of the apices of

Drift I. and Drift II., and the angular distances λ, λ_I, λ_{II} of the centre of the region from these apices. The positions of the apices used in calculating the above position angles and distances are :—

Solar Ant-apex	:	R.A. 90°	:	Dec. −34°
Apex of Drift I.	:	90°	:	−14°
„ Drift II.	:	288°	:	−64°.

As in the previous papers, the material available has been insufficient to enable the magnitudes of the stars (exclusive of the B.D. stars) to be determined satisfactorily, but an approximate value of the limiting magnitude (Harvard Scale) has been derived for each region from Van Rhijn's Table IV. (*Groningen Publications*, 27).

§ 4. *Region I.* (Groombridge 34).—For this region we have :—

Galactic Latitude	= − 18°
Position Angle of Ant-apex	= 117°·5
„ „ apex of Drift I.	= 102°
„ „ „ Drift II.	= 210°
λ, λ_I, λ_{II}	= 111°, 97°, 122° respectively.
Limiting Magnitude (H.S.)	= 12^m·5.

Fig. 1 shows the plot of the relative centennial proper motions. The scale is such that stars with absolute proper motions of about 5″ per century and over would not be represented on the diagram ; to bring them in, I have given them half their absolute proper motions and indicated their motions thus : ⊙. The origins of relative and absolute proper motions, R and A, are shown.

The meridian proper motions, derived from various sources, of 8 stars in the field (excluding Groombridge 34) are given by R. Schorr (*Eigenbewegungs-Lexikon*). Let P_x, P_y be the components of these proper motions (in arc per century) parallel to the equator and in declination : if M_x, M_y are the components of relative proper motion derived from the measures, the co-ordinates ξ, η are given by :—

$$\xi = M_x - P_x \quad : \quad \eta = M_y - P_y.$$

The following table gives the results for these stars :—

B.D.	B.D. Mag.	P_x.	P_y.	M_x.	M_y.	ξ.	η.
+43° 33	6·7	−3″·8	−1″·4	−0″·3	−1″·8	+3″·5	−0″·4
53	8·2	−4·7	−2·4	+0·8	−0·2	+5·5	+2·2
54	7·8	−2·2	−3·0	−2·2	−1·4	0·0	+1·6
42 41	6·0	+7·7	−3·9	+1·1	−0·5	−6·6	+3·4
48	5·9	+2·9	−0·4	+0·1	−1·2	−2·8	−0·8
53	8·9	−3·7	−1·2	−4·4	−1·7	−0·7	−0·5
57	8·0	+0·1	−5·0	−2·5	−0·5	−2·6	+4·5
61	8·3	+1·4	+0·7	−0·8	−1·1	−2·2	−1·8

The agreement in the last two columns is not very good, due possibly in part to the difficulty of deriving reliable photographic proper motions of the brighter stars in this list. The means for all the 8 stars are—

$$\xi = -0''·7 \quad : \quad \eta = 1''·0.$$

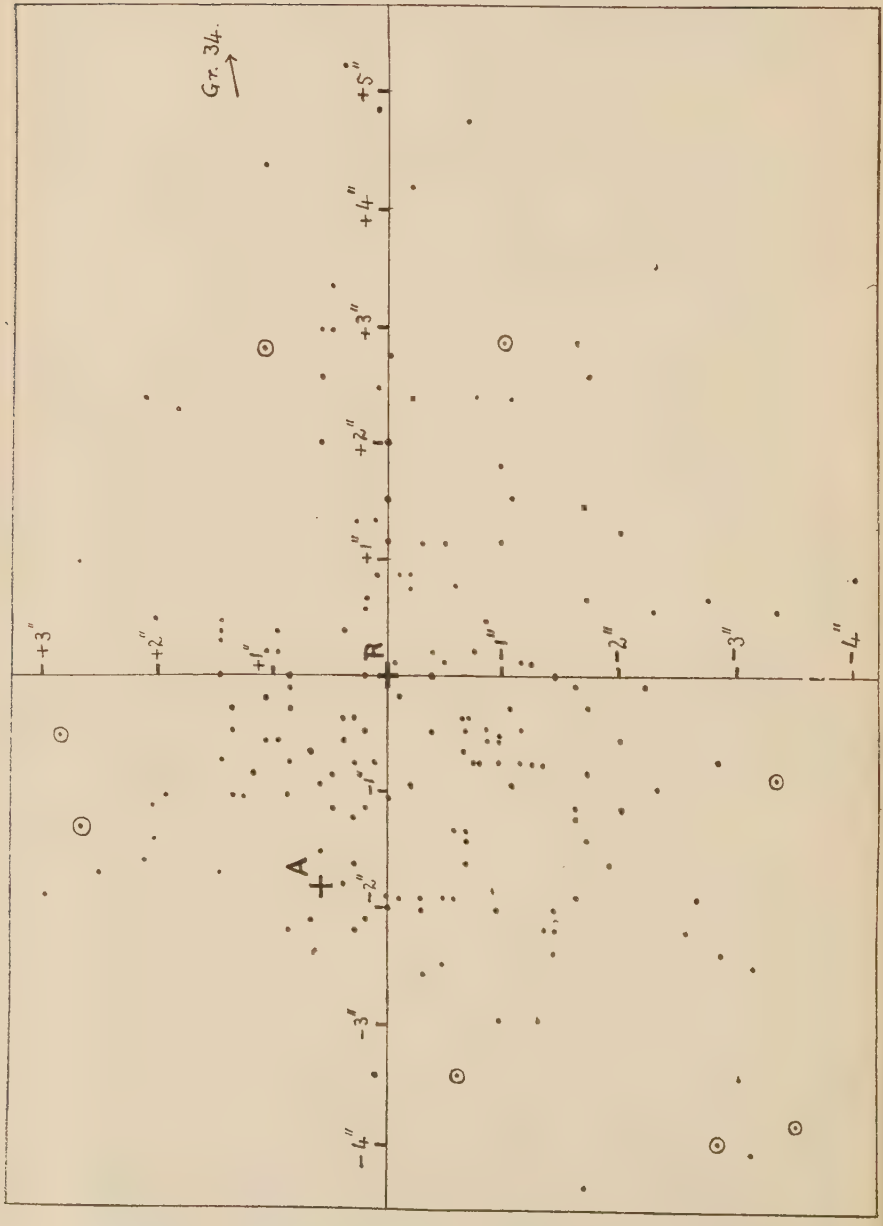


Fig. 1.—(Groombridge 34).

Altogether there are 44 B.D. stars in the field. The parallactic motions of these give :—

$$\xi = -2''\cdot0 \quad : \quad \eta = +0''\cdot5.$$

Excluding a group of the brightest stars—7 in number, and mean magnitude 6·6—the values are

$$\xi = -1''\cdot8 \quad : \quad \eta = +0''\cdot5.$$

The difference is slight. In deriving the absolute proper motions I have adopted the values :—

$$\xi = -1''\cdot8 \quad : \quad +0''\cdot6.$$

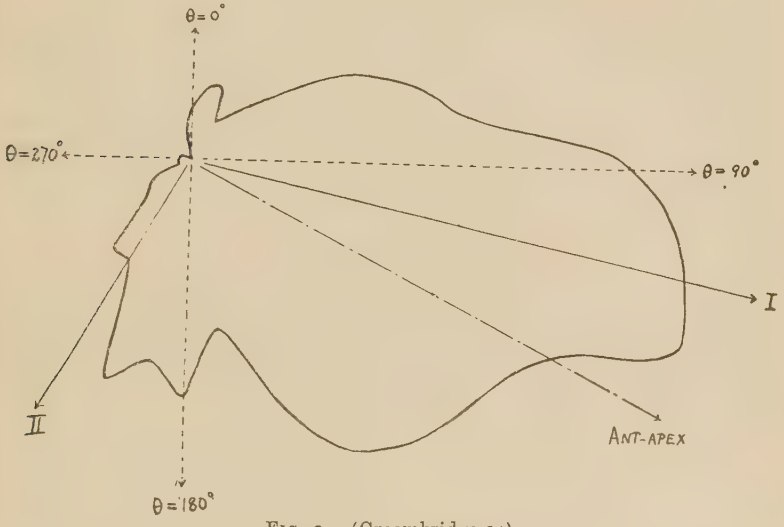


FIG. 2.—(Groombridge 34).

Fig. 2 shows the distribution of the proper motions of the 174 stars in the region (Groombridge 34 and its companion have been counted as one star). The direction of Drift I. agrees with that calculated for the brighter stars. Drift II. is in evidence but is not quite so pronounced as Drift I. There is an obvious extension of the curve in position angle 150° approximately. This is mainly due to a group of 17 stars with absolute proper motions $+1''\cdot5$, $-1''\cdot5$ per century (see fig. 1). The proper motions of these stars are small, and whether any physical significance can be attached to this feature of the curve it is somewhat difficult from the evidence to say.

The proper motion of Groombridge 34 has been determined by various observers. Burnham gives a list of determinations in "Measures of Proper Motion Stars" (Carnegie Institution of Washington, 1913). The most recent determination is that of *Cincinnati Publications*, 19 (see also *L.O.B.*, 344); the components of proper motion, given there, parallel to the equator and in declination, are

$$+286''\cdot2 \quad \text{and} \quad +39''\cdot5 \text{ per century.}$$

The photographic measures of Groombridge 34 (A) and its companion (B) show unmistakable orbital motion of the system. The components M_x, M_y of the relative centennial proper motions of both A and B with the mean epochs of the comparison plates are given here.

A.	Wt.	M_x.	M_y.	B.	M_x.	M_y.
		″	″		″	″
1904·98 / 1922·97	1	+290·2	+43·7			
1904·98 / 1923·90	2	+289·0	+44·7	1904·98 / 1923·90	+292·0	38·9
Mean		+289·4	+44·4			

According to Russell (*M.N.*, **67**, 135), the co-ordinates of B relative to A are +32″·3, +21″·6 for 1904·98, or in polar co-ordinates 38″·5, 56°. The motion of B relative to A in 100 years is +2″·6, −5″·5 or 6″·5 in position angle 155°—almost at right angles to AB; 99° to be exact. The period of the system would thus appear to be about 3600 years, if we assume a circular orbit and $i = 90°$.

Applying the values of ξ, η, we derive the proper motion of Groombridge 34:—

$$+291''{\cdot}2 \quad \text{and} \quad 43''{\cdot}8.$$

These values include the orbital motion of Groombridge 34 about the centre of gravity of A and B, but without precise information as to the ratio of the masses of A and B, this is the most that we can do at the present.

The following 10 stars (the approximate positions are given for 1900·0) have appreciable proper motions:—

	$\mu\alpha$.	$\mu\delta$.
	s	″
B.D. 42° 49	+0·56	+0·5
h m ° ′		
0 10·4, +43 45	+0·40	−7·0
11·1, 43 3	+0·84	−3·2
11·4, 42 50	+0·56	−1·3
11·6, 42 58	+0·83	+0·9
14·8, 44 14	+0·17	−7·8
15·8, 43 33	+0·63	−0·1
16·4, 42 55	+0·60	−0·5
16·7, 43 41	−0·37	−8·2
16·8, 44 9	+0·23	+4·5

§ 4. *Region II.* (β Persei).—For this region we have:—

Galactic Latitude	= − 14°
Position Angle of Ant-apex	= 144°
,, ,, apex of Drift I.	= 133°
,, ,, ,, Drift II.	= 215°
λ, λ_I, λ_{II}	= 85°, 68°, 138° respectively.
Limiting Magnitude (H.S.)	= $11^m{\cdot}5$.

FIG. 3.—(β Persei).

Fig. 3 shows the individual centennial proper motions of the 78 stars measured; also the origins R and A. β Persei is the only star in the field for which I could find a meridian proper motion, but as this star was not measured (except for one pair of plates—the colour screen having been used in both cases) the values of ξ and η have been derived entirely from the parallactic motions of 35 B.D. stars. The values are:—

$$\xi = -1''.0 \quad : \quad \eta = +1''.0 \text{ (per century).}$$

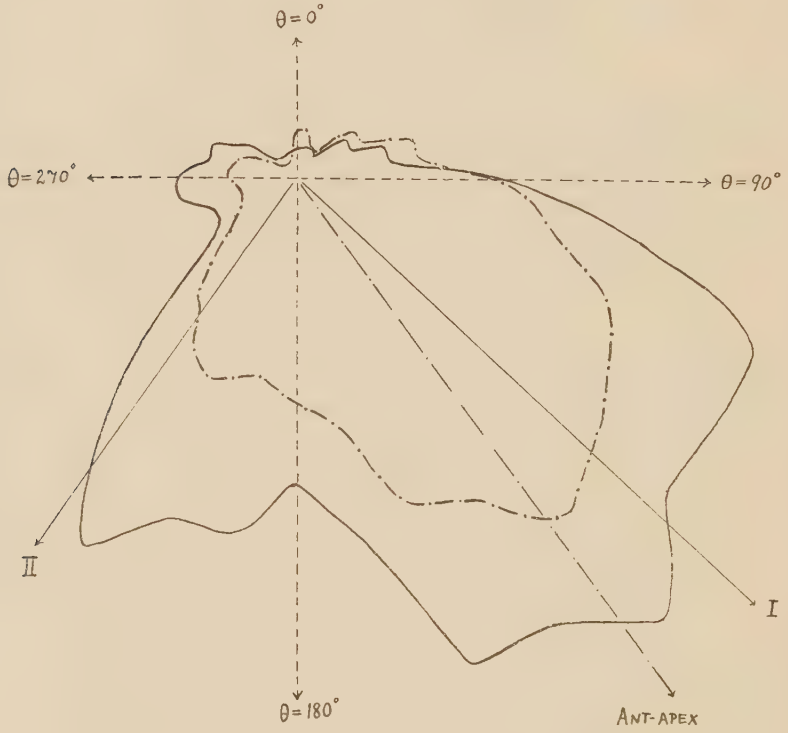


FIG. 4.

Curve ————. β Persei.
Curve —— · ——. β Persei and ρ Persei combined.

Fig. 3 shows that all the proper motions are of moderate magnitude. In fig. 4 the distribution of the absolute proper motions is shown (full curve) for the stars round β Persei. The broken curve represents, on a different scale, the combination of the 78 stars round β Persei with the 65 stars round ρ Persei. The figure found for ρ Persei (*M.N.*, **84**, 126) was notably a single drift curve with no indication of a Drift II.* The curve for β Persei shows both drifts, although Drift II. is numerically much feebler than Drift I. The combined curve for the

* The direction of Drift II. for ρ Persei was erroneously drawn—it should be in position angle 212°.

143 stars of the two regions, of course, accentuates this feature. It may be mentioned that the values of ξ, η in the case of ρ Persei were $-0''\cdot8$ and $+1''\cdot0$—practically the same as the values found above for β Persei. The comparison stars were of course different in the two fields; they were, however, of approximately the same mean magnitude.

The largest proper motion is that of B.D. 41°628 (Mag. 9·5), for which

$$\mu_\alpha = +0^s\cdot56 \quad : \quad \mu_\delta = -2''\cdot0.$$

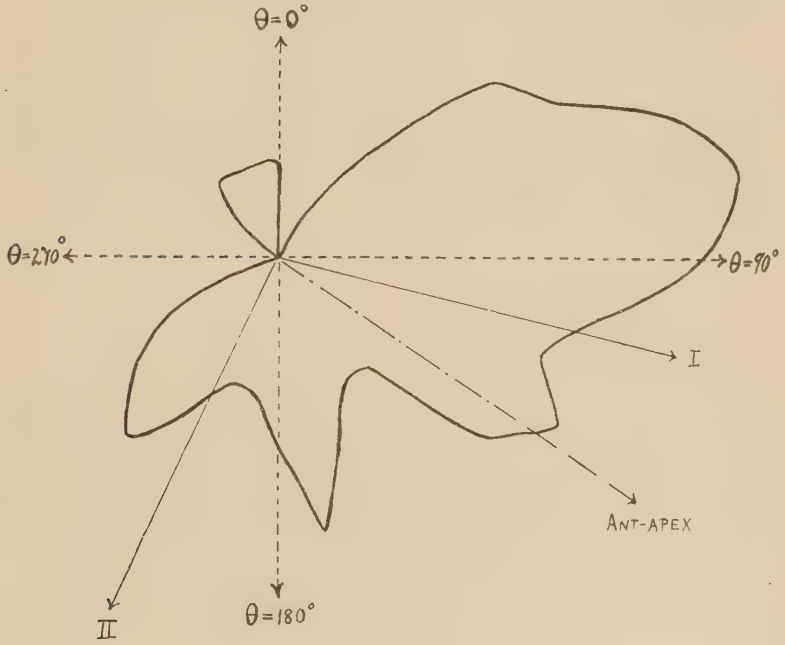


FIG. 5.—(Lalande 46650).

§ 5. *Region III.* (Lalande 46650).—For this region we have:—

Galactic Latitude	$= -56°$
Position Angle of Ant-apex	$= 124°$
,, ,, apex of Drift I.	$= 104°$
,, ,, ,, Drift II.	$= 204°$
λ, λ_I, λ_{II}	$= 95°$, $94°$, $82°$ respectively.
Limiting Magnitude (H.S.)	$= 12^m\cdot0$.

Fig. 5 shows the distribution of motion of the 42 stars in the region. The number of stars is hardly large enough to bring out the features of the motions successfully. However, the evidence seems strong enough to point to a systematic motion in position angle 80°—about 25° away from the direction of Drift I. as calculated for the brighter stars—but owing to the extreme sensitiveness of the diagram with regard to the

number of stars considered, it may not be advisable to put too much confidence in this result. The same difficulty arises with regard to Drift II.—the inclusion of one or two stars moving in position angle 200° would be sufficient to round off the re-entrant curve in this direction. On the whole, the diagram indicates systematic motion in directions very roughly those of the two drifts.

For two stars in the field meridian proper motions have been determined: (i) Lalande 46650 (*Cincinnati Publications*, 19) and (ii) B.D. 2°4723 (Munich 32805).

The values of μ_α and μ_δ for these stars are:—

(i) $\mu_\alpha = +6^s\cdot67$: $\mu_\delta = -96''\cdot9$
(ii) $\mu_\alpha = +3^s\cdot12$: $\mu_\delta = +17''\cdot3$.

The values of ξ, η have been determined from the parallactic motions of 25 B.D. stars, and the two stars above furnish a further determination of ξ, η. The results are:—

	ξ.	η.
Parallactic Motions	$-1''\cdot4$	$+0''\cdot1$
Lalande 46650	$-0\cdot8$	$-0\cdot4$
B.D. 2°4723	$-0\cdot9$	$+3\cdot3$
Adopted Values	$-1\cdot2$	$+0\cdot2$

The agreement as far as ξ is concerned is excellent. The proper motion in declination of B.D. 2°4723 would appear to be considerably in error. The B.D. magnitude of the star is 8·3, and the separate photographic determinations of proper motion are in very good agreement (the probable errors are within 0″·5 per century in each co-ordinate).

The following 8 stars have noteworthy proper motions; the adopted values of ξ and η have been applied in each case to the relative proper motions to give the centennial proper motions (absolute):—

Star.	μ_α.	μ_δ.	B.D. Mag.
	s	″	
B.D. +2°4718	+0·47	+ 1·1	8·5
4720	+0·53	− 4·4	9·0
4723	+3·14	+20·4	8·3
+1°4774 (Lal. 46650)	+6·71	−97·5	8·7
+1°4775	+0·65	− 3·4	9·5
4784	−0·47	− 4·0	9·3
4787	+0·37	− 6·3	9·5
$23^h46^m\cdot9$; +2°1′ (1900)	+0·57	− 5·3	...

§ 6. *Magnitudes of the Proper Motions.*—The following table gives the number of stars in the three regions with values of μ between various limits, μ being the total absolute proper motion along the great circle.

Table III.

μ.	I.	II.	III.
0″·0 – 1″·0	21	13	9
1·0 – 2·0	52	31	8
2·0 – 3·0	43	18	8
3·0 – 4·0	26	11	6
4·0 – 5·0	16	3	2
5·0 – 6·0	4	2	1
6·0 –	12	0	8

The large proportion of stars in Region III., with proper motions exceeding 6″ per century, is to be noticed.

Observatory, Cambridge:
1924 *May* 6.

Real and Apparent Radial Velocities. By A. V. Douglas.

(*Communicated by Professor Eddington, F.R.S.*)

The complete Doppler formula obtained by Silberstein * from de Sitter's spacetime theory, into which no limitation whatever is introduced, the observing station and star being treated as two free particles and the equations integrated in their full generality, is as follows:—

$$D = \gamma\left[1 \pm \sqrt{1 - \frac{\cos^2 \frac{r}{R}}{\gamma^2}}\right] - 1$$

where D = complete spectral displacement.

$\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v_0^2}{c^2}}}$ always slightly greater than 1, not determinable theoretically, but only by observation of Nature.

r = distance to star from observer.

R = curvature invariant of spacetime.

This formula for the spectral shift resolves itself into two parts—(1) a term depending upon an individual characteristic of any star, v_0, its actual radial velocity at its nearest approach to the observer whether that occurred in the past or will occur in the future, (2) a term depending upon the ratio $\dfrac{r}{R}$ and thus due to the four dimensional nature of elliptical spacetime.

* *Nature*, 1924 March 8, p. 350.

Although these two terms are inseparably amalgamated, yet for stars near to the observer it is evident that the first term dominates the result, while for very remote celestial objects the second term completely outweighs the first.

Assuming Silberstein's value of R as 4×10^7 parsecs and r to lie between 100 and 1000 parsecs, then term (2) lies between 0·7 and 7 km. per sec., and may thus be not merely comparable to but even equal to or greater than the true v_0-velocity.

In the light of the above it seems advisable to reconsider the work which has been done with regard to the progression of velocity with absolute magnitude.

W. S. Adams * showed that for K and M giants the radial velocity and its progression with absolute magnitude were greater than the tangential velocity and its progression; while for the giants of all types (F to M) taken together, the reverse was the case, the radial velocity and its progression being the smaller.

Eddington † showed for the G_8 to K_2 giants, when due allowance was made for the systematic effect arising from accidental errors in the spectroscopic determinations of absolute magnitude and parallax, and when using the value $\pm 0^m{\cdot}3$ for the probable error of the absolute magnitudes, that the progression derived from the proper motion data became comparable with, though not quite as great as, that obtained by Adams for radial velocities. If, however, the value $\pm 0^m{\cdot}4$, given by Adams ‡ as the probable error, was used, the resulting progression for the tangential velocity was more than doubled.

The question which Silberstein's work suggests is that the progression derived from radial velocities is not a true progression. It is influenced by the unduly high velocities of the remote stars, due solely to their distances, which stars in the catalogue are for the most part of high luminosity. Since the true v_0-velocities of those stars, which are both very bright and very remote, should be less than the values assigned to them, it is evident that the progression for true radial velocities should be greater than that given.

The list of spectroscopic parallaxes § used in both the above-mentioned investigations is made up, to the extent of perhaps 20 per cent. of its total, of stars distant between 100 and 1000 parsecs, and as has been pointed out this distance may cause the r/R shift to equal or outweigh the shift due to true velocity. Thus 20 per cent. of the stars involved in the investigation—and these belonging chiefly to the most luminous groups—would have their velocities reduced to varying extents, and it seems probable that this would produce a considerable change in the factor b in Adams' formula $\log v = a + bm$.

It would be interesting if this brought b up to such a value that it represented a progression comparable with that given by Eddington's formula $v = a + bm$ for tangential velocity, when the probable error of absolute magnitudes is taken nearer the value $\pm 0^m{\cdot}4$ given by Adams.

* Adams, Strömberg, and Joy, *Ap. J.*, **54**, 1921 (Table III., p. 9).
† Eddington and Douglas, *M.N. R.A.S.*, 1923 January.
‡ Adams, Joy, Strömberg, and Burwell, *Ap. J.*, **53**, p. 15, 1921.
§ *Ap. J.*, **53**, 1921.

But it was pointed out by Eddington that there is little hope of determining this probable error precisely, and therefore no reliable progression can be obtained from proper motion data with which to compare the progression for radial velocity in an attempt to work backward to an average value of v_0. If a large number of values of v_0 were calculated, it seems as though they alone would provide the material from which the progression of velocity with absolute magnitude might be determined.

M'Gill University, Montreal:
1924 *April* 11.

Mean Absolute Magnitude of a Group of Stars: Note on a Paper by Messrs. Young and Harper. By Gorakh Prasad, M.Sc.

(*Communicated by the Astronomer Royal for Scotland.*)

The absolute magnitudes and parallaxes, together with the methods employed for finding them, for stars of types F to M have been published by the Mount Wilson,* the Norman Lockyer, † and the Dominion Astrophysical ‡ Observatories. The method employed in the last paper for establishing the curves relating intensity differences of pairs of lines in the spectra of stars with absolute magnitude differs from the methods employed in the other two papers. One is struck by the elaborateness of the procedure adopted by Young and Harper and the comparative poorness of the result obtained. A casual glance at the sample figure § given by these authors with those given by Rimmer || suffices to demonstrate the far greater reliability of the latter curves. The plotted points in the figure of Young and Harper are so scattered that drawing a curve through them is highly arbitrary. No doubt a part of this scattering is due to the use of groups of stars instead of individual stars, and the procedure employed to make up for the consequent lack of points; but, as will appear from this note, some of it is due to the use of unsuitable formulæ for calculating mean absolute magnitudes. The correction found by the authors for the mean absolute magnitude computed from the trigonometrical parallaxes by their first method is

* W. S. Adams, A. H. Joy, G. Strömberg, and Cora G. Burwell, "The Parallaxes of 1646 Stars derived by the Spectroscopic Method," *Contributions from the Mount Wilson Observatory*, No. 199, and *Astrophys. J.*, 58, 1921.

† W. B. Rimmer, "The Luminosities and Parallaxes of 500 Stars, Types Fo to Mb," *Memoirs of the Royal Astronomical Soc.*, 62, 1923.

‡ R. K. Young and W. E. Harper, "The Absolute Magnitudes and Parallaxes of 1080 Stars," *Journal of the Royal Astr. Soc. of Canada*, 18, Nos. 1–2, 1924. (The complete results are not yet out.)

§ *Loc. cit.*, p. 29, fig. 8. It will be noted that the curve itself has not been shown in the figure. A curve drawn through the plotted points is described as a preliminary curve; but this does not minimise the importance of what is said here, as all the uncertainty of this curve enters into the final one.

|| *Loc. cit.*, p. 118, figs. 2 and 3.

wrong, and it is fortunate that the correction was not applied. The second method used by the authors introduces more serious errors.

1. *Correction due to the Accidental Errors in the Trigonometrical Parallaxes.*

Young and Harper have shown * that if the absolute magnitude of every star of a group is computed from its trigonometrical parallax and the mean of these absolute magnitudes is taken, the correction to the computed mean absolute magnitude due to accidental errors in the trigonometrical parallaxes is

$$-\frac{5h}{c^{1/2}}\int_0^\infty \frac{\log(1+x/\pi)+\log(1-x/\pi)}{10^{h^2x^2(0\cdot4343)}}dx, \quad . \quad . \quad . \quad (1)$$

where h is the modulus of precision of the trigonometrical parallaxes, c is the ratio of the circumference of a circle to its diameter, and π is the true parallax of the group of stars, if all the stars have the same parallax. The formula is stated to hold for a group of stars which do not have the same π and m, but it is not stated what meaning is to be attached to π in the expression (1) in this case. Apart from this, for values of x greater than π the integrand in (1) becomes imaginary, and for this reason alone (1) will cease to be applicable. In the most favourable case, when all the stars of the group have the same parallax π, which is so large that there is little likelihood of an accidental error being greater than it, we can replace (1) by

$$-\frac{5h}{c^{1/2}}\int_0^a \frac{\log(1+x/\pi)+\log(1-x/\pi)}{10^{h^2x^2(0\cdot4343)}}dx, \; 0<a<\pi,$$

and this objection disappears, but even in this case the formula is not applicable. The reason is this. The formula is based on the assumption that the stars of the group have the same true absolute magnitude M and the same π. The observed values of π, it is stated, will cluster about π, some being less and some greater. Now the stars are grouped together *according to the absolute magnitude computed from the observed* π. Hence a star whose observed parallax is appreciably different from π is put under a different group altogether. A little consideration will show that it is essential for the validity of the formula (1) that the stars of the group have the same *true* absolute magnitude. Since we have no means of knowing these, and all groups are necessarily based upon absolute magnitudes affected with errors caused by the errors in the trigonometrical parallaxes, formula (1) is of no practical use.

We can find as follows the excess of the true mean absolute magnitude over the computed mean absolute magnitude caused by the accidental errors in the trigonometrical parallaxes.†

Let the total number of stars whose trigonometrical parallaxes π_0 have been measured be N, where N is large. Assuming that the

* *Loc. cit.*, p. 38.

† The investigation by F. H. Seares, "A Troublesome Systematic Error" (*Monthly Notices*, **84**, 15, 1923), is not applicable to the present case.

frequency distribution of these stars as regards parallax is continuous, we can suppose that the number of stars found to have a measured parallax between π and $\pi+d\pi$ is $Ny(\pi)d\pi$. In consequence of the accidental errors in the measured parallaxes, the number, N_1, of stars having a true parallax between π and $\pi+d\pi$ is not $Ny(\pi)d\pi$ but $Nu(\pi)d\pi$, where

$$u(\pi)=y(\pi)-\frac{1}{4h^2}y''(\pi)+\frac{1}{32h^4}y^{iv}(\pi)+\ \ldots,$$

h being the modulus of precision of the measured parallaxes.*

On account of the accidental errors, the N_1 stars with a true parallax between π and $\pi+d\pi$ contribute a number, N_2, to the group of stars with a measured parallax between π_1 and $\pi_1+d\pi_1$, where

$$N_2=Nu(\pi)d\pi hc^{-1/2}e^{-h^2(\pi-\pi_1)^2}d\pi_1.$$

The error in the measured parallax of these N_2 stars is $\pi-\pi_1$ (in the sense true—measured). Out of the N stars, let the total number of stars having a measured parallax (say π_1') between π_1 and $\pi_1+d\pi_1$ be N_3. The number of such stars out of these N_3 stars as have a spectral type S and an apparent magnitude m satisfying the inequality

$$M\leqq m+5+5\log_{10}\pi_1',<M+dM$$

will obviously depend on π_1, S, and M. Hence we can assume for the number of such stars the expression

$$N_3f(\pi_1,\ S,\ M)dM.$$

Since the absolute magnitude, defined as the apparent magnitude at a distance corresponding to a parallax of 0″·1 (10 parsecs) is given by

$$M=m+5+5\log_{10}\pi, \qquad . \quad . \quad . \quad . \quad (2)$$

these stars have a computed absolute magnitude between M and M + dM. Hence out of the N_2 stars the number of such stars as have a spectral type S and a computed absolute magnitude between M and M + dM is N_4, where

$$N_4=N_2f(\pi_1,\ S,\ M)dM.$$

Differentiating equation (2), we see that an error $\pi-\pi_1$ in the parallax produces an error $k\ (\pi-\pi_1)/\pi$ in the absolute magnitude, where $k=2\cdot17$. This expression is valid only when $\pi-\pi_1$ is small compared with π. In general the error is

$$5(\log\pi-\log\pi_1).$$

The error in the sum, Σ M, of the computed absolute magnitudes due to errors in the parallaxes of the N_4 stars is therefore

$$5N_4(\log\pi-\log\pi_1).$$

* Eddington, *Monthly Notices*, **73**, 359, 1913, or Whittaker and Robinson, *The Calculus of Observations*, p. 207.

The total error in Σ M, obtained by integrating this, is

$$5\mathrm{N}hc^{-1/2}d\mathrm{M}\int_a^\infty\int_o^\infty u(\pi)e^{-h^2(\pi-\pi_1)^2}f(\pi_1,\mathrm{S},\mathrm{M})(\log\pi-\log\pi_1)d\pi d\pi_1,$$

where a is the smallest parallax used for the computation of an absolute magnitude.

The error in the mean absolute magnitude is therefore

$$\frac{5\mathrm{N}hc^{-1/2}d\mathrm{M}\int_a^\infty\int_o^\infty u(\pi)e^{-h^2(\pi-\pi_1)^2}f(\pi_1,\mathrm{S},\mathrm{M})(\log\pi-\log\pi_1)d\pi d\pi_1}{\mathrm{N}hc^{-1/2}d\mathrm{M}\int_a^\infty\int_o^\infty u(\pi)e^{-h^2(\pi-\pi_1)^2}f(\pi_1,\mathrm{S},\mathrm{M})d\pi d\pi_1}. \quad (3)$$

Since $u(\pi)$ is zero in the neighbourhood of $\pi=0$ and $\pi=\infty$, the range of integration 0 to ∞ may be replaced by α to β; $\alpha>0$, β finite. The sign of the correction will depend upon the functions u and f. If we assume for the moment that the variation of $f(\pi_1, \mathrm{S}, \mathrm{M})$ with π_1 is negligible, *i.e.* that there is approximately the same percentage of stars of spectral type S and absolute magnitude M in every group of stars formed according to the measured parallax, we see that the expression (3) is zero if $u(\pi)=0$ for $\pi<a$, and is some constant for other values of π. This shows that the correction chiefly arises from the unequal frequency distribution of the stars as regards parallax. Generally, $u(\pi)$ increases as π diminishes. This, it is easy to see, will make the integral in the numerator of (3) negative. The computed mean absolute magnitude will thus be greater than the true value, or in other words, the stars will be computed more dwarfish than they really are. The effect of the variation of a and $f(\pi_1, \mathrm{S}, \mathrm{M})$ can now be considered. An increase in a, that is, leaving out some more of the small parallaxes will make the correction increase by a negative amount. If $u(\pi)$ is a constant, a decrease of $f(\pi_1, \mathrm{S}, \mathrm{M})$ as π_1 increases will make the correction positive, an increase of $f(\pi_1, \mathrm{S}, \mathrm{M})$ as π_1 increases will make it negative. Hence, in general, for a group of stars in which the same proportion of large and small parallaxes occur as in the complete list, the correction will be negative. The correction for a group of stars which contains an excess of large parallaxes will also be negative, while for a group which contains an excess of small parallaxes the correction may be negative, zero, or positive. Usually it will be negative, but in very special cases, for groups in which only small parallaxes occur, the correction may be positive. These conclusions are contradictory to those of Young and Harper, who find that the correction is always positive, irrespective of the distribution of stars in the list between the different parallaxes, and that it is larger for a group of stars with small parallaxes.

2. *A Numerical Example.*

Since the list of stars used by Young and Harper is not yet available, we shall apply formula (3) to the list of stars used by Rimmer and, as it is necessary to neglect the small and negative parallaxes, we

shall assume that stars whose trigonometrical parallaxes are less than 0″·005 are not used. The distribution of stars as regards parallax is as follows:—

Parallax	″·005 – ″·015	″·015 – ″·025	″·025 – ″·035	″·035 – ″·045	″·045 – ″·055	″·055 – ″·065
Number of stars	39	50	40	26	27	15

Parallax	″·065 – ″·075	″·075 – ″·125	>″·125
Number of stars	14	32	14

Suppose that the absolute magnitude of each star is computed from its trigonometrical parallax and we pick out those stars which have a given spectral type, say G6 to G7, and an absolute magnitude between M and M+dM (say, to fix ideas, between −1·0 and +1·0), the group being such that in it the parallaxes are distributed in approximately the same way as in the complete list. We shall compute the correction for such a group. Let the number of stars in the group be n. Then, since the total number of stars is 257, we can put $f(\pi_1, \mathrm{S}, \mathrm{M})d\mathrm{M} = n/257$. The difference between $u(\pi)$ and $y(\pi)$ is small and will be neglected and h will be taken to be 48, corresponding to a probable error of 0″·01 in the trigonometrical parallaxes. We notice that the denominator in the expression (3) is equal to the number of stars in the group and is therefore n. The integral in the numerator can be evaluated by some method of numerical integration. Employing the method which corresponds to the Trapezoidal rule for evaluating single integrals,* we find that the numerator is equal to −·07 n. Hence the correction to be added to the computed mean absolute magnitude is −·07. This may be compared with the correction as found by Young and Harper. The average parallax of the group will be about the same as the average parallax of all the stars. Omitting the few stars which have parallaxes greater than 0″·125 and which contribute practically nothing to the correction of −·07 magnitude found above, the average parallax of all the stars is found to be 0″·04, and the correction according to Young and Harper † will therefore be about +0·2 magnitude.

To illustrate the effect of the variation of f, consider the group of 9 stars of type G6 and G7 with computed absolute magnitudes ·9, ·7, ·6, ·6, ·5, ·4, ·3, −·6, −1·0. These stars have parallaxes all of which are very nearly 0″·02. Here $2f(\pi_1, \mathrm{S}, \mathrm{M}) = 9/257$ in the neighbourhood of $\pi_1 = ″·02$ and is equal to zero at all other places. Performing the computation afresh, we find that the correction is a little less than +0·01 magnitude.

In case f is constant, we can find an approximate value of the correction by using the expression $k(\pi - \pi_1)/\pi$ instead of $\log\pi - \log\pi_1$. The correction computed by this method will not be very far out, since the small values of π are left out in computing absolute magnitudes, and the large values of the error $\pi - \pi_1$ are multiplied by a very small factor

* A. C. Aitkin and G. L. Frewin, "The Numerical Evaluation of Double Integrals," *Proc. Edinburgh Mathematical Soc.*, **42**, 4, 1924.

† *Loc. cit.*, p. 38.

$e^{-h^2(\pi-\pi_1)^2}$. When this is done we can perform the integration with respect to π_1 analytically and the integration with respect to π can then be performed numerically. The correction by this method for the above group of n stars comes out to be $-0\cdot12$ magnitude.

3. *Mean Absolute Magnitude from the Mean Parallax.*

A second method of computing the mean absolute magnitude used by Young and Harper consists in taking

$$5 + \overline{m} + 5 \log \bar{\pi}_0$$

as the mean absolute magnitude, the bars in the above expression indicating mean values. They state the correction to the computed absolute magnitude to be

$$5\,\overline{\log \pi_0} - 5 \log \bar{\pi}_0.$$

The correction, obviously, should be

$$5\,\overline{\log \pi} - 5 \log \bar{\pi}_0. \quad . \quad . \quad . \quad . \quad (4)$$

Of the errors in the computed mean absolute magnitudes, we must carefully distinguish between those which are due to the accidental errors in the trigonometrical parallaxes and those due to an inadequate method of computing the mean. The former will vanish if all the π_0's be replaced by π's, the latter will not vanish by this change. The correction calculated in Art. 1 to the mean of the separately computed absolute magnitudes will vanish if the accidental errors vanish. The correction to the mean absolute magnitude computed from the mean of the parallaxes does not vanish if the accidental errors all become zero. The expression (4) becomes in this case

$$5\,\overline{\log \pi} - 5 \log \bar{\pi}.$$

This will be large if the range in the parallaxes π is large. This is illustrated by the example considered by the authors. The correction found,* on the assumption that the use of spectroscopic parallaxes in place of the true parallaxes for the computation of this correction is justifiable, is $-0\cdot60$ magnitude for a group of 13 stars with a range in the spectroscopic parallaxes from $0''\cdot009$ to $0''\cdot100$. The correction is reduced to $-0\cdot14$ magnitude if one star is omitted, reducing the range in the parallaxes to $0''\cdot009$ to $0''\cdot029$. The correction will be very large if one of the parallaxes π is very small. A star with a negative measured parallax has presumably a very small true parallax. Hence we conclude that this method of calculating the mean absolute magnitude from the mean trigonometrical parallax is not good for the following reasons:—

1. Against the advantage of being able to include negative parallaxes must be considered the disadvantage of a large and uncertain error introduced by the method of computation itself. (The

* *Loc. cit.*, p. 40.

correction (4) cannot be calculated, for the true parallaxes π are unknown.)

2. The inclusion of only those stars which have a reasonably small dispersion in M and m and hence in π, as has been recommended by the authors, has the disadvantage of further curtailing the already small number of stars in a group, and also of bringing together in certain groups an excess of stars with only positive or only negative accidental errors in the trigonometrical parallaxes.

The statement that the true mean absolute magnitude of a group lies between the values found by the first and second methods is now seen to be invalid, and therefore there seems to be no reason for using the second method.

None of these objections apply to the formula used by Edwards in his recent paper * on the spectroscopic parallaxes of B type stars :—

$$\overline{M} = 5 + 5 \log \{n/\Sigma 10^{-0.2m}\} + 5 \log \bar{\pi}_0.$$

Here n is the number of stars in the group, which is formed according to the intensities of certain lines in the spectra. Each star has thus approximately the same absolute magnitude, but no restriction is placed on the dispersion in m or π.

In conclusion, I wish to express my gratitude to Professor Sampson for his kind interest in the preparation of this note.

The Maxima of Absorption Lines in Stellar Spectra (Second Paper).

By R. H. Fowler and E. A. Milne.

(*Communicated by the Director of the Solar Physics Observatory.*)

§ 1. In a previous paper † we have applied the theory of high-temperature ionisation, first developed by Saha, to the positions of maximum intensity of absorption lines in the stellar sequence of spectra. During the past year the series relationship of lines belonging to trebly-ionised silicon and to ionised carbon, which are important in the spectra of the hotter stars, have been discovered by A. Fowler.‡ It is the object of the present paper to use A. Fowler's results to calculate by the method of maxima the temperatures and pressures in the reversing layers of the hotter stars. We can supplement these results by provisional estimates for certain other spectra. We deal also with a number of other points which arise in applications of the theory.

§ 2. *Résumé of the Method.*—We use the terms "principal series" or "principal lines" to denote series or lines which are absorbed in transitions from the normal state of a neutral, singly-ionised, or multiply-ionised atom to an excited state, or state of higher energy. We use the

* *Monthly Notices*, **84**, No. 5, 367, 1924 March.
† *M.N.*, **83**, 403 (1923).
‡ *Proc. Roy. Soc.*, A, **103**, 413 (1923); A, **105**, 299 (1924).

terms "subordinate series" or "subordinate lines" to denote series or lines which are absorbed in transitions from one excited state to another excited state. In the usual spectroscopic terminology a principal series is one in which the constant term is derived from an orbit which is most conveniently described as having the azimuthal quantum number unity. Subordinate series are those whose constant terms are derived from orbits of higher azimuthal quantum numbers. It frequently happens that the state of lowest energy is one of azimuthal quantum number unity, and the two terminologies more or less coincide; this was in fact the case with all the atoms whose spectra were considered in our previous paper. But for ionised carbon, for example, the states of lowest energy are the pair $1\pi_{1,2}$, which give rise to the sharp and diffuse series, whilst the first σ state, which gives rise to the "principal series" of optics and has the azimuthal quantum number unity, is one of greater energy. In stellar applications we require a special name for lines derived from the state of lowest energy, and we shall therefore use the word principal in the sense stated above. When we wish to refer to a "principal series" in the optical sense, we shall call it a "first principal series," a "second principal series," etc., as the case may be. A second principal series is necessarily a subordinate series in our terminology, since it is derived from the *second* level of azimuthal quantum number unity.

In a mass of gas in statistical equilibrium, the number of neutral atoms in the normal state steadily decreases as the temperature rises, owing to increasing ionisation. The number of neutral atoms in a given excited state increases to a maximum and then decreases. It follows that if the gas is producing absorption lines, the intensity of a principal line will steadily decrease as the temperature increases, but the intensity of a subordinate line will rise to a maximum and then decrease.

We showed in our previous paper that if χ_1 is the ionisation potential of the atom, $\chi_1^{(r)}$ the (negative) energy of a given excited state, then for a given partial pressure P_e of free electrons, the temperature T of maximum concentration of atoms in the given excited state $\chi_1^{(r)}$ is given by

$$P_e = \frac{\chi_1^{(r)} + \frac{5}{2}kT}{\chi_1 - \chi_1^{(r)}} \cdot \frac{(2\pi m)^{\frac{3}{2}}(kT)^{\frac{5}{2}}\sigma_1}{h^3 b_1(T)} \cdot e^{-\chi_1/kT}. \quad . \quad . \quad (1)$$

In (1) m is the mass of the electron, k is Boltzmann's constant, h is Planck's constant, σ_1 is the symmetry number of the neutral atom, and $b_1(T)$ is the partition function, given by

$$b_1(T) = q_1 + q_1^{(2)} e^{-(\chi_1 - \chi_1^{(2)})/kT} + q_1^{(3)} e^{-(\chi_1 - \chi_1^{(3)})/kT} + \ldots \quad . \quad (2)$$

In (2) the q's are the weights of the corresponding stationary states, and the summation is extended over all the stationary states of which the given atom, in its neutral state, is capable. Equation (1) is an equation in T whose root is T_{max}, but we found it convenient to assume values of T_{max} and calculate from (1) the corresponding values of P_e. A certain difficulty connected with the convergence of the series in (2) has recently been removed by the work of H. C. Urey.* We return to this in § 9.

* *Astrophys. Journ.*, 59, 1 (1924).

The principal lines of the singly-ionised atom will also have a maximum as the temperature increases. We showed that the temperature at which the concentration of singly-ionised atoms reaches a maximum is given by

$$P_e = \left[\frac{\sigma_1\sigma_2}{b_1(T)b_2(T)}\right]^{\frac{3}{2}}\left[\frac{\chi_2+\frac{5}{2}kT}{\chi_1+\frac{5}{2}kT}\right]^{\frac{3}{2}}\frac{(2\pi m)^{\frac{3}{2}}(kT)^{\frac{5}{2}}}{h^3}e^{-\frac{1}{kT}(\chi_1+\chi_2)}. \quad (3)$$

where χ_2 is the second-stage ionisation potential and σ_2 and $b_2(T)$ refer to the singly-ionised atom.

The subordinate lines of the singly-ionised atom will have a maximum which arises in the same way as the maximum of those of the neutral atom. Provided that the first-, second-, and third-stage ionisation potentials are sufficiently well separated, so that only two stages of ionisation are simultaneously present in appreciable proportions, the maximum for subordinate lines of the singly-ionised atom is given by (1), when χ_1 and $\chi_1^{(r)}$ are replaced by the corresponding quantities χ_2 and $\chi_2^{(s)}$.

Similar formulæ hold for multiply-ionised atoms under the same conditions. If, however, an atom possesses a pair of consecutive ionisation potentials which are close together, more than two stages of ionisation may be present simultaneously, and a correction may be required. This is worked out in § 8, since at one time it was thought probable that it would be required for the discussion of trebly-ionised silicon.

We made the assumption, in our previous paper, that the maximum intensity of a line in the stellar sequence of spectra occurred when the concentration of atoms in the state capable of absorbing the line, calculated thermodynamically, had a maximum for given P_e. We assumed implicitly, therefore, that the reversing layer of a star could be regarded as in thermodynamic equilibrium, and that the state of affairs could be represented by introducing a partial electron-pressure P_e and a temperature T. These quantities must be taken to represent the mean values of the actual temperature and partial electron-pressure throughout the thickness of the layer in which the absorption line originates. It will be noted that we assumed P_e constant for different temperatures. To determine T_{max} from (1) the value of P_e must be known. We assumed the value $P_e = 1{\cdot}31 \times 10^{-4}$ atmos., this being the value which gave $T_{max} = 10{,}000°$ for the Balmer lines, which are observed to have their maximum in stars of type Ao. The significance of these assumptions will be discussed further in §§ 6, 7.

§ 3. *The Observed Maxima in Early-Type Stars.*—The following table shows the absorption-line maxima which occur in stars of type Ao and earlier. It has been compiled from information supplied by Mr. Baxandall, supplemented by H. H. Plaskett's chart of intensities in his memoir * on "Three O-Type Stars." The horizontal rows correspond to the spectral types, the vertical columns to the different elements. Against each maximum is inserted the ionisation potential for the removal of the next electron from the corresponding atom, when known. Thus "Si IV 44·9" means that here lines of the spectrum Si IV, due to the

* *Pub. Domin. Astrophys. Obs.*, **1**, 356 (1922).

Si^{+++} atom,* reach a maximum, and that the ionisation potential for the removal of the electron is 44·9 volts. An ionisation potential enclosed in brackets denotes a value somewhat less exactly known.

TABLE I.

Absorption-Line Maxima in Early-Type Stars.

Element. / Type	Hydrogen.	Magnesium.	Helium.	Carbon.	Silicon.	Nitrogen.	Oxygen.	Sulphur.
?				C IV (64)				
O5			He II 54·2					
O7						N III		
O9							O III	
O9–Bo				C III	Si IV 44·9			
B1					Si III		O II	S III
B2			He I 24·5	C II 24·3		N II		
B8					Si II			S II
Ao	H 13·5	Mg II 15·0						

One or two comments on the table may be made. Plaskett's chart of intensities shows the Pickering lines of the atom He^+ (part of the spectrum He II) to be still increasing in intensity at type O5. However, the intensities are estimated with reference to hydrogen, which is decreasing, so that it seems reasonable to put the maximum of He II at O5. Plaskett himself appears to place the maximum not far beyond O5, as he defines the hypothetical class Oo by the *disappearance* of the Pickering lines. The lines of Si IV are shown as having a maximum in type Bo in Plaskett's chart, though in the text he gives O9. The lines of Si IV and C III appear to have maxima approximately simultaneously. Bright lines probably belonging to C IV have been found by Merton † and identified with emission lines in the spectra of Wolf-Rayet stars, and we have therefore left a possible place for an absorption-line maximum for lines of C IV. Plaskett gives a maximum ‡ for N III in type O7, and, according to the Harvard observers,§ the lines rise to a great intensity in the star 29 Canis Majoris, of type Oe. The relative behaviour of nitrogen and oxygen thus appears anomalous if the assignment of lines to spectra is correct.

For the estimate of the ionisation potential of C IV and for other provisional data, in particular for C III and Si III, we are indebted to some unpublished investigations kindly put at our disposal by Mr. D. R. Hartree, of St. John's College, Cambridge.

Since this paper was written, Miss Payne ‖ has published a reinvestigation of the intensity curves of lines of Si I, II, III, and IV as a function of spectral type. Her curves show maxima of Si I in G5,

* It appears most suitable to speak of a *spectrum* as ZI, ZII, etc., and of the corresponding *atom* which emits it as Z, Z^+, etc. We attempt to do so consistently here.

† *Proc. Roy. Soc.*, A, **91**, 498 (1915).

‡ He allots the doublet λλ4097, 4103, to N^+, but A. Fowler in his "Report on Series in Line Spectra" gives reasons for allotting it to N^{++}, *i.e.* N III.

§ *H.A.*, **28**, pt. II., 233. ‖ *Harvard Coll. Obs.*, Circ. No. 252.

Si II in Ao, Si III in B2 – B1, and Si IV in Oe5 (or beyond). The resulting changes in Table I., if made, lead to obvious slight modifications in the deductions of this paper, which we have thought unnecessary to incorporate at this stage. The general agreement between theory and observation is hardly affected; if anything, it is improved. This paper contains certain results which may prove of considerable interest in future, for the observed and theoretical decay of the lines on the high-temperature side of their maxima seem to agree better than we had anticipated. It is to be hoped that these investigations will be extended to the C, N, and O lines of high-temperature stars.

§ 4. *Calculated Maxima.*—The calculation of the position of a maximum requires a knowledge of the appropriate ionisation potential χ and the energy-level $\chi^{(r)}$ from which the given subordinate series is derived. But a glance at formula (1) shows that the *order of magnitude* of P_e for given T_{max}, or of T_{max} for given P_e, is fixed almost entirely by the ionisation potential χ; provided we are sure the lines in question belong to a subordinate series, *i.e.* provided we know that $\chi^{(r)}$ does not coincide with χ, it is immaterial as regards order of magnitude what value we select for $\chi^{(r)}$. It is therefore possible to make rough calculations of maxima in certain cases even if the lines have not been fully classified, given an estimate of χ. We can thus first give the precise calculations for C II and Si IV based on A. Fowler's work. We can also legitimately use Table I., in view of the dominant effect of χ and the agreement in the values of χ in each row both here and at lower temperatures, to make estimates of χ for spectra as yet incompletely known. In this way we may possibly be able to make stellar observations help in the exact elucidation of these spectra, and complete our knowledge of the temperature scale for hot stars.

We arrange the data and calculations in the same way as in our previous paper. It may be recalled that $(2\pi m)^{\frac{3}{2}}k^{\frac{5}{2}}h^{-3} = 0{\cdot}332$; that kT is 0·860 volts when T = 10,000°; that wave numbers are converted into volts by multiplying by $1{\cdot}234 \times 10^{-4}$; and that pressures in dynes/cm.^{2} (as given by (1) and (3)) are converted into atmospheres by multiplying by 10^{-6}. More precisely we should say that we *define* our "atmosphere" to be 10^6 dynes/cm.^{2}.

(i.) *C II* (C^+) *Fundamental Series.* λλ4267·3, 4267·0 ($2\delta, {}_2 - 3\phi$). $1\pi_2 = 196670$, $\chi_2 = 24{\cdot}28$; $2\delta = 51108$, $\chi_2^{(2\delta)} = 6{\cdot}31$; $b_2(T) = 2$, $\sigma_2 = 1$.—We take $b_2(T) = 2$ because the $1\pi_1$-state is so close to the state of lowest energy $1\pi_2$ that they contribute equally to $b_2(T)$ near T_{max}. We take $\sigma = 1$ because the series electron is a single electron with a normal 2_2 orbit. (The other electrons are 2 in 1_1 and 2 in 2_1 orbits.) The doublet 4267 is precisely analogous to λ4481 Mg II, and is observed to have a maximum in stars of type B2 simultaneously with the spectrum He I. It should be noted that the ionisation potentials of the atoms He and C^+ (24·5, 24·3) are almost identical. We find

$T_{max} = 14{,}000°$,	$P_e = 3{\cdot}58 \times 10^{-6}$ atmos.
$T_{max} = 15{,}000°$,	$P_e = 1{\cdot}71 \times 10^{-5}$ atmos.
$T_{max} = 16{,}000°$,	$P_e = 6{\cdot}47 \times 10^{-5}$ atmos.
$T_{max} = 17{,}000°$,	$P_e = 2{\cdot}23 \times 10^{-4}$ atmos.

(ii.) *Si IV* (Si^{+++}). *Second Principal Doublets.* λλ4088, 4116 ($2\sigma - 2\pi_{1,2}$). $1\sigma = 364117$, $\chi_4 = 44{\cdot}94$; $2\sigma = 170105$, $\chi_4^{(2\sigma)} = 20{\cdot}98$; $b_4(T) = 1$, $\sigma_4 = 1$.—The electronic structure is that of Na. This well-known doublet has been classified by A. Fowler. It is observed to have a maximum at Bo or O9 simultaneously with the C III triplet λλ4647, 4650, 4652. Assuming that χ_3 and χ_1 are sufficiently far apart for no correction (§ 8) to be required (this is not yet completely certain), we find from (1)

$T_{max} = 23{,}000°$, $P_e = 3{\cdot}92 \times 10^{-6}$ atmos.
$T_{max} = 25{,}000°$, $P_e = 3{\cdot}03 \times 10^{-5}$ atmos.
$T_{max} = 27{,}000°$, $P_e = 1{\cdot}75 \times 10^{-4}$ atmos.

(iii.) *Speculations on C III and Si III.*—If we may so far trust our theory, we must expect that the ionisation potential of C^{++} will be about 45 volts. We may also rely on the value 64 volts for C^{+++}, though this has not yet been fixed exactly. From these values and the known value 45 volts for Si^{+++}, we can derive a rough value for Si^{++}. The argument is that, as a matter of empirical fact, the ratios of two consecutive ionisation potentials for the removal of two electrons in equivalent orbits are roughly the same for all atoms of the same external structure. This ratio is known for Mg, Ca, Sr, Ba, Zn, Cd. We find the values 0·508, 0·515, 0·515, 0·520, 0·514, 0·516. These are all atoms with two equivalent series electrons in n_1-quantum orbits. We may therefore expect the same equality of the pairs C^{++} C^{+++} and Si^{++} Si^{+++}, though not necessarily the same ratio (·51) owing to the different net charge. Since $45/64 = 0{\cdot}703$, we expect that $\chi_3 = (0{\cdot}703)45 = 31{\cdot}6$ for Si^{++}. If the slight increase of the ratio with atomic number is significant, χ_3 should lie between 32 and 33.

We could confirm this estimate if we could estimate χ_2 for O^+. Now χ_1 for O has recently been properly determined.* Presumably O has 2 equivalent series electrons in 2_2 (circular) orbits. The only atom of similar structure which we might legitimately use for comparison is He, for which the ratio $\chi_1/\chi_2 = 0{\cdot}452$. This gives $\chi_2 = 30$ for O^+. Very little weight can be attached to these calculations for O, but so far as they go they confirm the estimate that the ionisation potential typical of maxima in B1 stars is about 32 volts.

The lines observed in stars are λλ4647, 4650, 4652 C III, and λλ4552, 4567, 4574 Si III. They are almost certainly homologous lines, type $2s–2p_{1,2,3}$. For provisional calculations one may take the rough values $\chi_3^{(2s)} = 18$ (C III), 15 (Si III); $b_3(T) = 1$, $\sigma_3 = 2$. This indicates that for $P_e = 1{\cdot}31 \times 10^{-4}$ atmos., the maximum of the lines Si III, $\chi_3 = 32$, should occur at about 19,000°.

We put forward these speculations leading to 32 volts for the ionisation potential of Si^{++} for what they are worth. We had been informed by A. Fowler that the 1S term for Si III was still somewhat uncertain, but that a value near 256,000 (31·6 volts) was perhaps to be expected. The independent line of argument here presented seems to us to point quite strongly to some such value.

* Birge, *Journ. Opt. Soc. Amer.*, **8**, 233 (1924), and Hopfield, *Phys. Rev.*, **21**, 710 (1923), assign the ionisation potentials 13·55 and 10·31 volts to O and S.

We are also informed by Mr. D. R. Hartree that an extrapolation by his method from Mg and Al^+ leads to about 31 volts for Si^{++}.

§ 5. *The Temperatures and Pressures in the Reversing Layers of the Hotter Stars.*—Logarithmic interpolation amongst the values of P_e calculated in the preceding section gives the following values of T_{max} for $P_e = 1{\cdot}31 \times 10^{-4}$ atmos. These form a continuation of the table on p. 420 of our previous paper. The value for He I has been repeated for

TABLE II.

Temperatures of Early-Type Stars.

$T_{max.}$	Type.	Maximum of
16,000°	B2	He I ; 1π–$m\delta$, etc.
16,560°	B2	C II ; 2δ–$m\phi$.
(19,000°	B1	Si III ; O II.)
26,600°	Bo–O9	Si IV ; 2σ–2π.
35,200°	? O5	He II ; "4686" and Pickering series.

convenience, recalculated for the correct ionisation potential 24·5 volts, and that for He II has been added also. The value in brackets is based on a provisional estimate only. C III is omitted, as its observed coincidence with Si IV has been used to derive the value for Si III.

We noted in our previous paper that the method of maxima indicates a temperature-scale which agrees with the observed temperatures for the late-type stars, but gives much higher temperatures for early-type stars than those commonly attributed to them. It is at once apparent that this indication is continued by the results for carbon and silicon. Indeed, the maximum of Si IV in stars of type Bo–O9 assigns to them a temperature of 26,600°, far higher than had been anticipated. A change of temperature of 10,000° between stars of types B2 and Bo–O9 seems at first sight unlikely. It is therefore desirable to examine in detail the exact meaning of our determinations.

A glance at Table I., wherever the ionisation potentials are known, shows that the spectra of atoms with about the same ionisation potential have simultaneous maxima. As examples we have H and Mg II in Ao, and He I and C II in B2. This is in accord with theory, since, as we have seen, the position of a maximum for a given P_e depends mainly on χ and comparatively little on $\chi^{(r)}$, $b(T)$, and σ. Now the ionisation potentials in the row B2 are 24 volts and in the row Bo–O9 are 45 volts. It is improbable that any small change of temperature could be responsible for such a large change of excitation as is implied by a change of relevant ionisation potential from 24 volts to 45 volts. Quite apart from the details of our method, therefore, A. Fowler's determination of the ionisation potential of Si^{+++} as 45 volts, combined with the observed maximum of the Si IV doublet in stars of type O9–Bo, would have suggested that the temperature-step from B2 to Bo must be a large one. The same argument can be applied to the jump of 6000° from Ao to B2. Roughly speaking, the paramount circumstance is the value of the ionisation potential, and the temperature of the maximum is

approximately proportional to the ionisation potential in the absence of any other alteration of circumstances.

We have already made one fundamental application of the observed maxima of Table I. on this basis, in assigning 45 volts to C III. It is possible to use Table I. to estimate roughly other unknown ionisation potentials. For example, by itself it strongly suggests that the ionisation potentials of Si^+ and S^+ are 18–19 volts, that of N^+ 24 volts, that of S^{++} 32 volts, and those of O^{++} and N^{++} between 48 and 53 volts. The circumstance that the ionisation potentials (and calculated temperatures) increase apparently erratically with spectral type need cause us no concern, for the steps of the subdivisions of spectral type are purely arbitrary.

The temperatures given in Table II. depend on the assumed partial electron pressure $P_e = 1{\cdot}31 \times 10^{-4}$ atmos. Though the large increase in ionisation potential from He and C^+ to C^{++} and Si^{+++} necessarily implies a considerable step in temperature from B2 to Bo, the actual amount of the step will be smaller if a smaller value of P_e is required for the maxima of C III and Si IV than we have assumed. Thus for Si IV, $P_e = 3{\cdot}9 \times 10^{-6}$ atmos. gives $T_{max} = 23{,}000°$. $T_{max} = 20{,}000°$ requires a value of P_e near 8×10^{-8} atmos.

For ϵ Orionis, the typical star of type Bo, Coblentz* found a temperature of 13,000°–14,000° from thermocouple observations. For ϵ Persei, also of type Bo, H. H. Plaskett† has given the provisional temperature of 15,000° as determined by the wedge method. For γ Cassiopeiæ, of type Bop, Plaskett has found 15,000° and Sampson‡ 16,900°. The temperatures determined by the method of maxima can only be reconciled with these spectrophotometric temperatures by taking values of P_e 10^6 times smaller than those indicated by the observed maxima in types M to Ao.

Plaskett§ has directed attention to certain anomalies in the intensity-distribution in the spectra of O-type stars. The Victoria observers find that the O-type stars as a class are white. They appear almost as white as stars of type Ao, whilst they would be expected to be bluer than the B-stars. It is possible, therefore, as Plaskett suggests, that the intensity-distributions in the spectra of the hotter stars fail to give an indication of their temperatures; in other words, that owing to departure from the black body distribution the colour-temperature has no relation to the true temperature. If this is so, the objections to the high temperatures determined by the method of maxima disappear.

It must, however, be repeated that the adoption of a constant value for P_e is a pure assumption, made provisionally as a working hypothesis. The adopted value $P_e = 1{\cdot}31 \times 10^{-4}$ atmos. gives a temperature-scale from M to Ao, and probably to B2, which is in agreement with the spectrophotometric scales. It may be that for high ionisation potentials smaller values of P_e are required.

It should be observed in passing that the spectrophotometric scales give the effective temperature of the star. The reversing layer should be at a lower temperature—its average temperature should be in the

* *Pop. Astron.*, **31**, 113 (1923).
† *Pub. Domin. Astrophys. Obs.*, **2**, 258 (1923).
‡ *M.N.*, **83**, 174 (1923). § *Loc. cit.*

neighbourhood of, or somewhat lower than, the Schwarzschild boundary temperature, which is some 15–20 per cent. lower than the effective temperature. Perhaps we ought to adopt 9000° for the reversing-layer temperature of stars of type Ao. This would require a value of P_e in the neighbourhood of 10^{-5} atmos. We do not propose, however, to consider this modification further in this paper.

§ 6. *The Meaning of* P_e.—The physical significance of the value found by the method of maxima for the partial electron pressure P_e has already been briefly described by one of us elsewhere.* The value found corresponds to the level in the stellar atmosphere at which a strong subordinate-series line originates. The greater the absorption coefficient of the gas as a whole, in the frequency concerned, the less far can we see into it, and the nearer the surface must be the level in which the emergent radiation originates, and so the lower the pressure. Now the absorption coefficient of a gas like hydrogen, for example for the Balmer lines, is proportional to the product of the concentration of atoms in the 2-quantum state and the atomic absorption coefficient. The method of maxima ascertains when the concentration of atoms in the 2-quantum state is a maximum for given P_e, but it is clear that the greater the atomic absorption coefficient the smaller will be the appropriate value of P_e. If the atomic absorption coefficients for different atoms are different, the corresponding pressures will be different. If atoms with high ionisation potentials have large atomic absorption coefficients for certain lines, we should be entitled to take lower values of P_e for these lines in the hotter stars, and the calculated temperatures would be lower. It would be out of place to enter into details, but it may be mentioned that though the theory of atom-structure suggests the possibility of some such effect, the actual amount appears to be too small to have any serious influence on the value of P_e.

The next point is the ratio of the number of atoms of the kind concerned, at any level, to the number of free electrons. An element less abundant relative to free electrons will yield lines orginating at greater depths, and so at higher electron pressures, than an element more abundant. This does not invalidate our previous statement that the method of maxima avoids certain difficulties in Saha's method of marginal appearances introduced by the question of the relative abundance of the elements. In the method of marginal appearances, the relative abundance of the elements comes in even if the partial electron pressure is regarded as known—the lines of a more abundant element will appear earlier and linger longer than those of a less abundant one. In the method of maxima, the question of relative abundance does not enter if P_e can be regarded as known. We are simply pointing out here that the proper value of P_e must be a function of the abundance of the atom in question relative to free electrons.

For the hotter stars, where owing to greater ionisation free electrons will be more abundant relative to atoms, we should expect to require larger electron pressures than for the cooler stars. The effect would only give rise to a small numerical factor. Such as it is, however, it would lead to higher values of T_{max} than we have calculated.

* Milne, *Phil. Mag.*, 47, 211 (1924); *Proc. London Phys. Soc.*, 36, 109 (1924).

A third point is the dependence of P_e on g, the surface value of gravity. For the same "optical depth," P_e would be expected to vary roughly proportionally to g. The argument is too well known to repeat. Dwarf stars should therefore require larger values of P_e than giants, and maxima in dwarfs should occur at higher temperatures than in giants. This is in agreement with observation, though the fact is usually expressed in the form that for given spectral type giants have lower effective temperatures than dwarfs. If g_1, g_2 are the values of gravity for a giant and dwarf of the same spectral type, T_1 and T_2 their temperatures, then assuming simultaneous maxima we have roughly

$$\frac{T_1^{\frac{5}{2}} e^{-\chi/kT_1}}{g_1} = \frac{T_2^{\frac{5}{2}} e^{-\chi/kT_2}}{g_2} \quad . \quad . \quad . \quad . \quad (4)$$

where χ is the mean effective ionisation potential for the lines whose maxima fix the type. Consider as an example the Sun (dwarf) and the brighter component of Capella (giant). We have $g_1 = 0{\cdot}067 \times 10^4$, $g_2 = 2{\cdot}73 \times 10^4$, $g_2/g_1 = 40{\cdot}7$. Let us take $T_2 = 6000°$, and use (4) to calculate T_1. From the data and calculations of our previous paper, a maximum of a subordinate line at a temperature of 6000° corresponds to about $\chi = 8$ volts, assuming P_e to be about 10^{-4} atmos. for the dwarf. We then find from (4), $T_1 = 5000°$. This should be the temperature of the reversing layer in Capella. The Mount Wilson colour-indices * would appear to indicate a temperature between 5000° and 5500°; Eddington † adopts 5500°. The calculated difference of temperature between giants and dwarfs is thus of the same order as that observed, and the theoretical dependence of P_e on g is therefore confirmed. ‡

A fourth point is the dependence of P_e on the excitation potential $\chi - \chi^{(r)}$, which represents the energy that has to be communicated to the atom in order to transfer it from the normal state to the state in which it can absorb the given subordinate series. We have seen that for given P_e, the value of T_{max} depends principally on χ, the ionisation potential, and hardly at all on $\chi^{(r)}$. But the bigger the excitation potential $\chi - \chi^{(r)}$, the smaller is the concentration of excited atoms, and so the smaller the absorption coefficient of the gas as a whole. Hence the greater is the depth at which the line originates, and the larger is P_e. Thus lines for which $\chi^{(r)}$ is relatively small compared with χ would be expected to require greater values of P_e.

It is of interest in this connection to tabulate the fraction of atoms in given excited states at maxima. We showed in our previous paper § that if x_1 is the fraction of neutral atoms, then at the maximum of a given subordinate series ($\chi^{(r)}$) of the neutral atom the value of x_1 is given by

$$[x_1]_{max} = \frac{\chi_1^{(r)} + \frac{5}{2}kT_{max}}{\chi_1 + \frac{5}{2}kT_{max}},$$

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* Seares, *Astrophy. Journ.*, **55**, 198 (1922). † *M.N.*, **84**, 109 (1924).

‡ Similar calculation has been made by J. Q. Stewart, *Pop. Astron.*, **31**, 88 (1923).

§ The notation has been changed slightly. What was called $(1 - x)$ in the previous paper is here called x_1.

and the fraction of atoms in the given excited state $\chi^{(r)}$ at this maximum is $n_1^{(r)}$, given by

$$[n_1^{(r)}]_{max} = x_1 \frac{q_1^{(r)} e^{-(\chi_1 - \chi_1^{(r)})/kT_{max}}}{b_1(T_{max})}.$$

Similar relations hold for maxima belonging to the singly- or multiply-ionised atom, on replacing the suffix 1 by 2, 3 . . ., provided the ionisation potentials are well separated. The following table gives the values of x and $n^{(r)}$ for the various maxima discussed in this and the previous paper, assuming $P_e = 1{\cdot}31 \times 10^{-4}$ atmos.

TABLE III.

Fractional Concentrations at Maxima.

Atom.	χ.	$\chi^{(r)}$.	T_{max}.	x_{max}.	$n^{(r)}{}_{max}$.
Na	5·12	3·03	3,900°	0·65	$1{\cdot}3 \times 10^{-3}$
Ca	6·08	4·20	4,270°	0·73	$4{\cdot}4 \times 10^{-3}$
Ca	6·08	3·16	4,420°	0·58	$2{\cdot}7 \times 10^{-4}$
Mn	7·39	5·29	5,020°	0·75	$6{\cdot}0 \times 10^{-3}$
Mg	7·61	4·92	5,250°	0·69	$1{\cdot}8 \times 10^{-3}$
Mg	7·61	3·29	5,440°	0·51	$4{\cdot}9 \times 10^{-5}$
H	13·54	3·39	10,000°	0·35	$7{\cdot}7 \times 10^{-6}$
Mg+	15·00	6·15	10,220°	0·49	$2{\cdot}1 \times 10^{-5}$
He	24·5	3·61	16,000°	0·25	$6{\cdot}4 \times 10^{-8}$
C+	24·28	6·32	16,560°	0·36	$5{\cdot}9 \times 10^{-7}$
[Si++	32	15	19,000°	0·53	$1{\cdot}6 \times 10^{-5}$]
Si+++	44·94	20·98	26,600°	0·53	$1{\cdot}5 \times 10^{-5}$
[C++	45	18	26,600°	0·47	$3{\cdot}5 \times 10^{-6}$]
He+	54·18	3·39	35,200°	0·18	$9{\cdot}1 \times 10^{-8}$

[ ] Provisional.

We commented on the small values of $n^{(r)}$ at maxima in discussing the peaks of the curves plotted in our previous paper. That only one atom in 10^5, for example, should be capable of absorbing a Balmer-line frequency, even at the maximum of these lines, seemed difficult to reconcile with the great intensity of the Balmer lines. Pannekoek * has called attention to the same point. But it is now known that atomic absorption coefficients are very large,† and these small values of $n^{(r)}$ at maxima no longer cause any difficulty.

Here we wish to draw attention to the variation in values of $n^{(r)}{}_{max}$ from atom to atom. From what has been said, other things being equal, we should expect to require bigger values of P_e the smaller the values of $n^{(r)}{}_{max}$. Increasing P_e increases T_{max}, and so increases

* *Physica*, 2, 293 (1922). † Milne, *Phil. Mag.*, 47, 209 (1924).

$n^{(r)}{}_{\max}$. Now there is no reason to expect that the values of $n^{(r)}{}_{\max}$ for different atoms should be all equal, but a glance at the table shows that exceptionally low values are given by He and He^+. This arises because for He and He^+ $\chi^{(r)}$ is exceptionally small compared with χ—a very high degree of excitation is necessary for these atoms before they are capable of absorbing the arc lines of helium and the Pickering lines respectively. This suggests that higher pressures should be adopted for He and He^+, thereby increasing the temperature of maxima. Type B2 would then have a temperature nearer to that found from Si IV for B0 ; and the temperature of maximum development of the Pickering lines would be increased to above 35,000°, thus giving more room for types O9 to O5.

We may summarise the results of this discussion by saying that the partial electron pressure at a maximum, which is the mean electron pressure at which the line originates, should increase with increasing values of gravity, and with increasing excitation potential $\chi - \chi^{(r)}$, and should decrease with increasing abundance of the element relative to free electrons, and with increasing atomic absorption coefficient. The mathematical formulation of the problem * depends on the structure assumed for the stellar atmosphere, but ultimately it should be possible to use the observed maxima to furnish information concerning atomic absorption coefficients, values of gravity, and relative abundances. As regards the hotter stars, the discussion gives no basis for suspecting serious errors in the high temperatures found by the method of maxima. Indeed, it suggests rather that the temperatures of maxima of He I and He II should be somewhat increased.

§ 7. *The Value of P_e for Principal Lines.*—For an intense principal line of a neutral atom, or for the maximum of a principal line of an ionised atom, the fraction of atoms in the required state is almost unity ; for the required state is here the normal state. On the other hand, we have just seen that at the maxima of subordinate lines the fraction of atoms in the required state is from 10^{-3} to 10^{-5}, at least for the lower ionisation potentials. Thus atoms in the required state are 10^3 to 10^5 times as abundant for intense principal lines as for intense subordinate lines. It follows that principal lines must originate at much higher levels in the stellar atmosphere than subordinate lines, and consequently at much smaller pressures. In the absence of precise calculations, it is reasonable to conjecture that the value of P_e for a principal line is from 10^{-3} to 10^{-5} of the value for a subordinate line.

This appears to be the explanation of the fact noted in our previous paper, that the observed maxima of the lines of Ca^+, Sr^+, and to some extent Ba^+, which are all principal lines, demanded lower values of P_e than 10^{-4} atmos. The maximum of a principal line of an ionised atom is necessarily much blunter than that for a subordinate line, and the spectral type for the maximum is correspondingly difficult to determine. But it is of interest to use the observed maxima as far as possible. We therefore give a recalculation of the results for Ca^+, Sr^+, and Ba^+. The values of P_e are calculated from formula (3).

* A preliminary result has been given in *Phil. Mag.*, **47**, 227 (1924).

Table IV.

Values of P_e (in Atmospheres) for Maxima of Principal Lines of Ionised Atoms.

$T_{max.}$	Ca^+. $\chi_1 = 6{\cdot}08$ $\chi_2 = 11{\cdot}82$	Sr^+. $\chi_1 = 5{\cdot}665$ $\chi_2 = 10{\cdot}97$	Ba^+. $\chi_1 = 5{\cdot}18$ $\chi_2 = 9{\cdot}95$
3500		$4{\cdot}6 \times 10^{-10}$	$5{\cdot}6 \times 10^{-9}$
4000	$3{\cdot}24 \times 10^{-9}$	$2{\cdot}03 \times 10^{-8}$	$1{\cdot}79 \times 10^{-7}$
4500	$7{\cdot}8 \times 10^{-8}$	$4{\cdot}0 \times 10^{-7}$	$2{\cdot}73 \times 10^{-6}$
5000	$1{\cdot}02 \times 10^{-6}$	$4{\cdot}4 \times 10^{-6}$	$2{\cdot}52 \times 10^{-5}$
5500	$8{\cdot}5 \times 10^{-6}$	$2{\cdot}62 \times 10^{-5}$	$1{\cdot}58 \times 10^{-4}$

If the H and K lines of Ca^+ are taken to have a maximum in stars of type Ko, of temperature 4500°, then from the table $P_e = 7{\cdot}8 \times 10^{-8}$ atmos. This is about 6×10^{-4} times the value for subordinate lines, $P_e = 1{\cdot}3 \times 10^{-4}$ atmos., in agreement with the above conjecture. The same pressure fits the observed maxima for Sr^+ and Ba^+, these having slightly lower temperatures.

The maximum of a principal line of an ionised atom thus appears to originate at a partial electron pressure of 10^{-7} atmos. to 10^{-8} atmos. Optically and thermodynamically the principal lines of a neutral atom should behave in the same way. The intensity of a principal line of a neutral atom increases steadily with decreasing temperature in the range of stellar temperatures and so has no maximum there. We may still conclude, however, that when its intensity is great it also originates at a partial electron pressure of 10^{-7} atmos. to 10^{-8} atmos.

We should mention here that there is no theoretical discontinuity between principal lines and subordinate lines, but that there is a practical one, since we do not actually find examples in stellar spectra of subordinate lines for which $\chi - \chi^{(r)}$ is very small compared with χ. Just as, however, exceptionally large values of $\chi - \chi^{(r)}$ lead to very small concentrations at maximum and so may require values of P_e *larger* than normal, exceptionally small values of $\chi - \chi^{(r)}$ mean large concentrations at maximum and values of P_e *smaller* than normal. If $\chi - \chi^{(r)}$ is very small, the concentrations and values of P_e approximate to those of a principal line. In Table III. values of $\chi - \chi^{(r)}$, somewhat small compared with χ, are found for Mn, Ca, Mg, and Na. It is probable that these lines require pressures smaller than $1{\cdot}31 \times 10^{-4}$ atmos. and correspondingly smaller T_{max}, but we shall not enter into details here.

§ 8. *Determination of Maxima with two Close Ionisation Potentials.* —When two ionisation potentials, χ_1 and χ_2 say, are close together, which means in practice that they have a *ratio* nearly 1, the analysis of our previous paper must be supplemented as follows. The work is written out for the first and second stages, and applies of course unchanged to any consecutive stages when the suffixes 1, 2, and 3 are replaced throughout by s, $s+1$, and $s+2$.

Let x_1, x_2, x_3 be the fractions of neutral, singly- and doubly-ionised atoms. Then *

$$\frac{x_2}{x_1}\mathrm{P}_e = \frac{(2\pi m)^{\frac{3}{2}}(k\mathrm{T})^{\frac{5}{2}}\sigma_1}{b_1(\mathrm{T})h^3}e^{-\chi_1/k\mathrm{T}}, \quad . \quad . \quad . \quad (5)$$

$$\frac{x_3}{x_2}\mathrm{P}_e = \frac{(2\pi m)^{\frac{3}{2}}(k\mathrm{T})^{\frac{5}{2}}\sigma_2}{b_2(\mathrm{T})h^3}e^{-\chi_2/k\mathrm{T}}, \quad . \quad . \quad . \quad (6)$$

$$x_1 + x_2 + x_3 = 1. \quad . \quad . \quad . \quad . \quad (7)$$

(i.) To determine the maximum of a subordinate line of the neutral atom we have to make $dn_1^{(r)}/d\mathrm{T} = 0$, where

$$n_1^{(r)} = x_1 q_1^{(r)} e^{-(\chi_1 - \chi_1^{(r)})/k\mathrm{T}}/b_1(\mathrm{T}),$$

and x_1 is calculated from (5), (6), and (7). Variations of $b_1(\mathrm{T})$ and $b_2(\mathrm{T})$ with temperature are ignored. The resulting equation is most conveniently written in the form

$$\mathrm{P}_e = \frac{\chi_1^{(r)} + \frac{5}{2}k\mathrm{T}}{\chi_1 - \chi_1^{(r)}} \frac{(2\pi m)^{\frac{3}{2}}(k\mathrm{T})^{\frac{5}{2}}\sigma_1 e^{-\chi_1/k\mathrm{T}}}{b_1(\mathrm{T})h^3} + \frac{1}{\mathrm{P}_e}\frac{\chi_2 + \chi_1^{(r)} + 5k\mathrm{T}}{\chi_1 - \chi_1^{(r)}} \frac{(2\pi m)^3(k\mathrm{T})^5\sigma_1\sigma_2 e^{-(\chi_1+\chi_2)/k\mathrm{T}}}{b_1(\mathrm{T})b_2(\mathrm{T})h^6} \quad . \quad (8)$$

The second term if small enough may be treated simply as a correcting term to P_e as calculated by (1). In other cases equation (8), a quadratic in P_e, must be solved directly. This equation must be used to calculate P_e whenever the ionisation potential of the stage in question is closely *followed* by the ionisation potential of the succeeding stage.

(ii.) To determine the maximum of a subordinate line of the ionised atom we have to make $dn_2^{(r)}/d\mathrm{T} = 0$, where

$$n_2^{(r)} = x_2 q_2^{(r)} e^{-(\chi_2 - \chi_2^{(r)})/k\mathrm{T}}/b_2(\mathrm{T}),$$

and x_2 is calculated from (5), (6), and (7). We find here

$$\mathrm{P}_e = \frac{\chi_2^{(r)} + \frac{5}{2}k\mathrm{T}}{\chi_2 - \chi_2^{(r)}} \frac{(2\pi m)^{\frac{3}{2}}(k\mathrm{T})^{\frac{5}{2}}\sigma_2 e^{-\chi_2/k\mathrm{T}}}{b_2(\mathrm{T})h^3} - \mathrm{P}_e^2 \frac{\chi_1 + \chi_2 - \chi_2^{(r)} + \frac{5}{2}k\mathrm{T}}{\chi_2 - \chi_2^{(r)}} \frac{b_1(\mathrm{T})h^3}{(2\pi m)^{\frac{3}{2}}(k\mathrm{T})^{\frac{5}{2}}\sigma_1} e^{\chi_1/k\mathrm{T}} \quad . \quad (9)$$

Equation (9) must be used whenever the ionisation potential of the stage in question is closely *preceded* by the ionisation potential of the preceding stage.

The corrections in (8) and (9) result in making the maxima for the lines of the two stages occur further apart in the temperature scale.

If we express the correction in the form of a factor $(1 + \alpha)$, multiplying the P_e given by (1), then α is of the order $e^{-(\chi_2 - \chi_1)/k\mathrm{T}_{\max}}$. Since

* These are equations (7) and (8) of our previous paper in the present more systematic notation.

kT_{max} varies roughly as χ_1 or χ_2, we see that the importance of the correction is determined by the closeness of χ_1/χ_2 to 1.

§ 9. *The Exact Form of* $b(T)$.—In our work hitherto we have ignored all questions as to the convergence of the series for $b(T)$, and have deliberately assumed that its numerical value in all cases of interest in reversing layer problems is that of its first term (but see also § 4, C II). The explicit form (2) is derived on the assumption that atoms in any stationary state behave practically like a perfect gas, and does not fulfil the assumed conditions unless the series really terminates for some reason after a finite (but possibly large) number of terms. Our assumption has recently been shown to be correct for reversing layer problems by an investigation due to H. C. Urey;* he resolves the difficulty in a manner which, in outline at least, we believe to be essentially correct.

Urey points out that atoms in any state, normal or excited, should probably be assigned a "volume" comparable to the volume swept by the orbit of their series electron. This is equivalent to assuming that in the ordinary collisions of statistical equilibrium the electronic orbits of two distinct atoms (or molecules) cannot in general interpenetrate. When these orbits are the quickly-traversed space-filling orbits of the normal atom, we have every reason to believe this assumption to be true. It must be just this non-mechanical impenetrability of the quantised orbits of two different atoms or molecules which gives to atoms and molecules (in fact to matter) the familiar property of extension. At first sight it is somewhat surprising that this property is to hold also for the large nearly degenerate orbits of highly excited atoms and even for the almost perfectly degenerate orbits of the excited hydrogen atom. But it must be borne in mind that the only necessary part of Urey's assumption is: *The orbits of the electrons of two distinct atoms can never interpenetrate in any collision which leads to no change in the quantum state of either atom.* When we are to consider any question of statistical (or thermodynamic) equilibrium, we may (and in fact must) properly exclude from consideration all actual processes of exchange, provided we retain merely the hypothesis that the necessary processes of exchange can occur occasionally. For the purpose of determining the equilibrium state, the "volume" of any atom, excited or not, is that region of space into which on the average no other body may come without causing a change of quantum state. To a first rough approximation this may very reasonably be taken to be the volume swept out by the orbit of the series electron, or, as Urey does, a sphere of radius equal to the semi-axis major of this orbit. Such "volumes" are then strictly comparable to the volumes of van der Waals' theory.

On this basis Urey has shown by thermodynamic arguments, which we need not repeat, that the weights $q^{(r)}$ in our formulæ must be replaced by functions $q^{(r)}/s^{(r)}$, where $s^{(r)}$ is a certain thermodynamic function depending on the "volume" of the atom in the r^{th} state. If for simplicity we assume that such atoms form a gas obeying van der Waals' equation,

$$p(v - b^{(r)}) = RT,$$

* *Loc. cit.*

it follows thermodynamically that

$$s^{(r)} = e^{\beta^{(r)}P/kT} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In (10) $\beta^{(r)}$ is four times the "volume" of the atom in state r and P is the *total* pressure. It is thus shown that our $b(T)$ should correctly be replaced by

$$u(T) = \Sigma_r q^{(r)} e^{-\beta^{(r)}P/kT - (\chi - \chi^{(r)})/kT} \quad . \quad . \quad . \quad (11)$$

A numerical discussion for the hydrogen atom is at this stage sufficient to illustrate the numerical order of $u(T)$ for all atoms. Urey's assumptions in this case lead to $\beta^{(r)}/k = 1{\cdot}863 \times 10^{-8} r^6$. The early terms are then all identical with those for a perfect gas, but the terms later than a certain stage are rapidly cut out and the series converges. We tabulate some additional calculations.

TABLE V.

$\frac{1}{2}u(T)$, *the Corrected Value of* $\frac{1}{2}b(T)$.

P atmos. / T°	10^{-3}.	10^{-6}.	10^{-9}.
10,000	1·0007	1·017	1·55
20,000	3·0	62	1900

The order of magnitude of P and P_e will generally be the same. We see at once that, for calculations of T_{max} at reversing layer pressures, $b(T)$ and $u(T)$ reduce to their first term and are indistinguishable. Our previous calculations are unaltered. Further, even for calculation of $n^{(r)}$ as a function of T the difference between $b(T)$ and $u(T)$ is of no account. The exact formula for $n^{(r)}$ reduces to

$$n^{(r)} = \frac{q^{(r)} e^{-\beta^{(r)}P/kT + \chi^{(r)}/kT}}{u(T)e^{\chi/kT} + 0{\cdot}332\,T^{\frac{5}{2}}/P_e} \quad . \quad . \quad . \quad (12)$$

If, for example, P, $P_e = 10^{-6}$ atmos., and $T = 20{,}000°$, $u(T)$ would have to be greater than 10^7 to affect $n^{(r)}$ appreciably. Thus the values 2 and 124 are indistinguishable. The values of $s^{(r)}$ for the first few values of r do not differ appreciably from 1, so that the extra term in the numerator of (12) may also be discarded for ordinary absorption series.

§ 10. *Manganese.*—Since our previous paper the term values for the spectrum Mn I have become fully available.* Certain lines of this spectrum are well known in stars, and we record here the calculations of maxima as an addendum to our previous paper. The prominent narrow triplet Mn λλ4031, 4033, 4034·5 is a principal line of type $1S - (mP + C)$, (Catalán), and its behaviour in the stars conforms to this classification, for it has no maximum. Two other Mn lines of stellar importance are λλ4018, 4041. These are the most prominent

* Catalán, *Phil. Trans.*, A, 223, 127 (1922).

lines of Multiplet VI. (Catalán), type D′ – D″. The complete multiplet consists of the 13-lines λλ4018, 36, 41, 49, 56, 60, 64, 68, 70, 79, 79, 83, 84. We have therefore

Mn. (D′D″) *multiplet.* λλ4018, 4041 (*and others*). $1S = 59937$, $\chi_1 = 7{\cdot}39$; $D_1' = 42885$, $\chi_1^{(D_1')} = 5{\cdot}29$; $b_1(T) = 1$, σ_1 *uncertain, assumed* 2. The maximum is difficult to determine, but is probably at Ko. A more complete examination of this maximum is desirable. We find

$$T_{max} = 4{,}500^\circ, \quad P_e = 1{\cdot}38 \times 10^{-5} \text{ atmos.}$$
$$T_{max} = 5{,}000^\circ, \quad P_e = 1{\cdot}22 \times 10^{-4} \text{ atmos.}$$
$$T_{max} = 5{,}500^\circ, \quad P_e = 7{\cdot}45 \times 10^{-4} \text{ atmos.}$$

The chief interest of Mn consists in the very large value of $\chi_1^{(D_1')}$. As a result, the value of $(n^{(r)})_{max}$ for Mn in Table III. is the largest we have yet found. The suggested T_{max} is high for type Ko, unless P_e is smaller than normal. But as we saw in § 7 this smallness of P_e is to be expected.

Other spectra are rapidly becoming available. The spectrum Cr I is now classified.* The only line whose stellar behaviour we have traced is λ4254 Cr, which is a principal line and shows no maximum.

§ 11. *Summary.*—As a result of the foregoing discussion we have found—

(i.) The temperature scale for early-type stars is roughly B2, 16,500°; B1, 19,000°; Bo – O9, 26,500; O5, more than 35,000°.

(ii.) The partial electron pressure P_e at maximum is the mean electron pressure at which a line originates. It is fixed by fixing T_{max} for that line. For other lines and conditions P_e increases with increasing values of gravity and increasing excitation potential $\chi - \chi^{(r)}$, and decreases with increasing abundance of the element relative to free electrons and with increasing atomic absorption coefficient.

(iii.) The calculated temperature difference between giants and dwarfs of the same spectral type agrees with observation.

(iv.) The excitation potential $\chi - \chi^{(r)}$ is exceptionally large compared with χ for both He and He^+. The fractional concentrations at maximum of atoms absorbing these lines is exceptionally small. Both P_e and T_{max} should probably be larger for these atoms than the values here assigned.

(v.) The concentration at maximum of normal ionised atoms capable of absorbing principal lines is exceptionally large (unity). For such lines probably $P_e = 10^{-7}$ atmos. – 10^{-8} atmos., and the observed maxima of the principal lines of Ca^+, Sr^+, and Ba^+ agree with these pressures.

In conclusion it is again a pleasure to thank Mr. Baxandall for the observational material which he has put at our disposal.

* Gieseler, *Zeit. f. Phys.*, **22**, 201 (1924); see also Laporte, *Zeit. f. Phys.*, **23**, 135 (1924), for a part at least of the spectrum Fe I; Goudsmit, *Nature*, **113**, 238 (1924).

Cumulative Ionisation in Stellar Atmospheres. By Martin C. Johnson.

1. The first quantitative relations between the temperature and pressure in a star's reversing layer and the degree of ionisation of the elements which give rise to its absorption lines, were given by M. N. Saha (*Phil. Mag.*, 1920 October and December, 1921 February; *Proc. Roy. Soc.*, A, 1921 May). The present note is concerned, not with the equations put forward by Saha and improved by Fowler and Milne (*M.N.R.A.S.*, 1923 May), but with the mechanism of the ionisation, in particular with the relation of cumulative or step-by-step ionisation to some apparent anomalies encountered in the application of the equations.

It is important to realise how far discussions of the equations and of the mechanism are independent. Milne has shown (*Observatory*, 1923 April, p. 123) that the equations of thermodynamic equilibrium on which the subject is based are perfectly general energy relations, and that their success cannot, therefore, be used in support of any one mechanism, *e.g.* photoelectric ionisation or electronic collisions. He has also shown, however (*Phil. Mag.*, 1924 January, p. 214) that this independence of mechanism is not incompatible with a certain general condition, namely, that the success of the equations requires the ratio of rates of recombination and ionisation to be simply proportional to the pressure of the gaseous assemblage. This implies a restriction on any mechanism that may be chosen for the gas in thermodynamic equilibrium, that it must carry with it a recombination mechanism providing the necessary relation to pressure.

But though the equilibrium equation alone thus leaves open alternatives of mechanism of ionisation, Fowler has calculated (*Phil. Mag.*, 1924 February) probabilities of effective collisions in a mass of gas in thermodynamic equilibrium: by relating this to Milne's expression (*Phil. Mag.*, 1924 January) for the probability of photoelectric ionisation under similar conditions, he concludes that in a typical stellar reversing layer at 10,000° abs. and an electron pressure of 10^{-4} atmosphere, for H atoms in a state to absorb Balmer lines, the ratio of the number of collision ionisations to radiative ionisations will be of the order 5×10^{-3}. The ratio will vary directly with the pressure if the other quantities are constant, *e.g.* it will show a greater efficacy of collision than radiative ionisation by the time a pressure of 10^{-1} atmosphere is reached.

If, therefore, the stellar reversing layer is in thermodynamic equilibrium the problem of the mechanism is soluble in spite of its independence of the original equation of Saha or of that of Fowler and Milne. If, however, we relax the condition of equilibrium by assuming that the reversing layer is subject to a flux of radiation passing through it from the photosphere below which is at a higher temperature, we have the condition which has been described by Russell (*Astroph. Journal*, 1922 May). Now Russell's work on this point is based on K. T. Compton's theory of cumulative ionisation (*Phil. Mag.*, 1922 March; *Phys. Rev.*, 1922 October), which, however, provides a ratio of collision excitation to radiative excitation inversely proportional to the square of the gas pressure. One purpose of this note is to investigate the source of the discrepancy between this ratio and the ratio obtained by Fowler (*loc. cit.*).

The first inquiry is, therefore, whether any possible difference of conditions in the gas can alter the relation to pressure of the ratio of collision to radiative ionisation. There are two of these differences of condition:

(i) The relaxation of strict thermodynamic equilibrium.
(ii) The difference between direct ionisation and cumulative ionisation.

If these physical differences do not adjust the discrepancy, the question next arises whether the Compton theory of cumulative ionisation is valid when applied to a stellar atmosphere.

Now Russell's application of Compton's theory is made to explain why Ba is more completely ionised in the sun than Na, in spite of the fact that the ionisation potentials of the two are almost the same. After discussing Compton's particular theory, therefore, some alternative explanations of the Ba anomaly are examined, and an attempt made to define the scope of cumulative ionisation as a theory to explain phenomena in stellar atmospheres.

2. (i) The new condition introduced in a star's reversing layer if thermodynamic equilibrium does not hold is solely that due to the photospheric radiation from a hotter region. At first sight, then, this can only increase radiative ionisation, *i.e.* the disturbance of equilibrium will not by itself affect the results of collisions as these depend on the temperature and the nature of the atoms only.

But given an added possibility of excitation of atoms by the radiation itself from the photosphere, certain collisions of electrons with these excited atoms will be effective in ionising, which in the state of strict equilibrium would have been inelastic or only effective as far as excitation. Similarly, collisions which in equilibrium resulted in exciting atoms which returned to normal state without ionisation may now begin a process which can be completed by the photospheric radiation or by the resonance radiation which is its secondary result. Thus the effectiveness of a collision may be greater when in the presence of the photospheric radiation than if it depended solely on the temperature of the gas in the enclosure of equilibrium theory, even though the number of collisions and the velocity of the electrons is unaffected. The number of effective absorptions of radiation under these conditions will still, however, be dependent on the concentration of atoms, and the number of two-body collisions will still depend on the square of the gas pressure; Compton's relation of the ratio of radiation to collision excitations is therefore not accounted for by the disturbance of equilibrium alone on this simple theory: nor will any fresh possibilities arise if the simple theory is supplemented by considering the superelastic collisions and also multiple transformations of radiation which occur in any actual atmosphere.

(ii) There are other differences in ionisation probability if cumulative as well as direct ionisation is active. At any fixed temperature and pressure the relative probability of ionisation of a series of elements will be in the same order as their ionisation potentials: the retarding effect on the completion of ionisation of an element due to the presence of mixtures of different gases can be ignored in this connection, as

it is the same for direct and cumulative processes; but the relative values of the resonance potentials may alter the order of ionisation of the elements if cumulative processes are predominant. In the case of He with ionisation potential 25·5 volts and resonance potentials at 20·4 and 21·2 volts, if the mean energy of thermal agitation is less than the lowest resonance potential, in this case less than about 0·8 of the ionisation potential, the collisions will be mainly inelastic. In the case of Mg with ionisation potential of 7·7 volts and resonance potentials at 2·6 and 4·4 volts, the separate steps by which ionisation could be effective do not involve more energy than about 0·4 of the ionisation potential: a fortunate series of collisions with electrons of this energy might make the gas appear as if its ionisation potential were lowered in a ratio considerably greater than could be the case with He.

Clearly, in considering these differences due to cumulative ionisation, two causes of increased ionisation are active:

(*a*) The number of distinct energy levels in the atom.

(*b*) The smallness of the fraction of the ionisation potential separating each of these levels.

These considerations can be extended to successive ionisations with similar dependence on the magnitude of the steps in the cumulative stages. Here, however, another variable of importance is the quantity usually referred to as "τ," or the time which elapses before an electron returns to its normal orbit after being displaced. This has been measured in certain cases (Foote and Mohler "Origin of Spectra," p. 93), and also calculated (Milne, *Proc. Phys. Soc.*, 1924 February), and seems to be of the order of 10^{-8} sec. for return after excitation, but of a larger order for recombination after ionisation (Foote and Mohler, p. 156). Extensions of any cumulative theory to multiple ionisation must take into account the fact that return of an electron after excitation depends only on the structure of the atom, whereas return after ionisation depends on the neighbouring material also.

The above differences between the probability of direct ionisation and cumulative ionisation depend on the value of energy levels in the atom, and are independent of the mechanism by which the electron is moved among these levels; if we consider the particular cases of radiative and collision excitation there is a further difference: take the case of an atom receiving more than sufficient energy to remove an electron from level A to level B but not enough to remove it to level C. If the form of energy of the medium can be represented as

$$W = h\nu$$

the atom will be entirely transparent to the monochromatic radiation of this frequency and the electronic orbit will not be affected. If, however, we consider the energy of the form

$$\tfrac{1}{2}mv^2$$

involving the velocity of an electron, this energy can be transferred and the atom will be excited, and its valency electron lifted to orbit B, the surplus energy going as kinetic energy to one of the colliding bodies. Given a certain amount of energy available, it has therefore a greater

chance of being communicated to the atoms if the mechanism is one of collision than if the mechanism is radiative; for the former it must involve a velocity greater than a definite minimum, for the latter it must involve an exact frequency such that the energy must not be above nor below the minimum. The case of excitation required for cumulative ionisation is here in contrast to that of direct ionisation; no such restriction is placed on the energy ionising photoelectrically: photoelectric experiments indicate that absorption occurs at all frequencies above the "threshold," the surplus energy going, as in the case of both excitation and ionisation in collisions, to the kinetic energy of the emitted electron.

To sum up: if we relax the condition of thermodynamic equilibrium, and also consider cumulative as well as direct ionisation, the probability of ionisation in a stellar reversing layer will be increased. This increase affects radiative excitation directly and collision excitation indirectly. The amount of the increase depends on the situation and number of the resonance potentials of an element, complicated in the case of multiple ionisation by the increase of the quantity "τ." In general, the extra energy is more available by collision than by radiation absorption. Other things being equal, however, the relation of each of these processes to gas pressure is still governed by the common-sense principle which is in agreement with the result calculated for the equilibrium case and for direct ionisation. The details of Compton's theory of cumulative ionisation must, therefore, be examined in view of its pressure relation before it is accepted as applicable to astrophysics.

3. The cumulative ionisation which Compton and Russell apply to the Saha theory is described by the former in a theory of the thermionic tube (*Phys. Rev.*, 1922 October). The conditions of the process there discussed are: An enclosed tube containing gas at a definite pressure, containing also a concentric cylindrical anode outside an incandescent cathode between which a definite accelerating potential is established by means of a battery. The application to the stellar case is made on the grounds of two quantities which in the case of the tube are measurable, the gas pressure and the potential difference between the electrodes. Consider the use of the former:—

Compton discusses the fractions of the total number of atoms of the gas in the tube that are partially ionised at any instant; if P_i be the fraction of their number excited by electronic impact and P_r the fraction excited by the absorption of resonance radiation from other excited atoms, then P_r will vary directly with the gas pressure p, as mentioned above (2 (i)). Compton, however, also finds that

$$P_i \propto \frac{I}{p}$$

which, combined with an expression for P_r, results in the ratio which is inconsistent with Fowler's equilibrium calculation and with the perfectly general considerations above, which do not depend on equilibrium.

Compton's complete equation is

$$P_i = \frac{n\tau}{v\mathrm{N}p}$$

where p is the gas pressure, N is the number of atoms in unit volume at unit pressure, n is the number of electrons emitted per second by the cathode, τ is the average duration of the excited state, and v is the volume of the region of the tube in which effective collisions occur.

Now, if the theory is to provide a relation of collision excitation to radiative excitation in a stellar atmosphere as, in fact, Compton uses it, the conditions under which these quantities were defined no longer hold; since instead of the emitting cathode there is a volume density of electrons varying with the total pressure of the gaseous assemblage, and instead of a fixed number of atoms in an enclosed tube there is an open atmosphere in equilibrium under gravitational and radiative forces. An attempt to transform each of the above factors when the argument is to be applied to the reversing layer of a star must end in the previous conclusion that will make

$$P_i \propto p^2.$$

Compton's view that as lower gas pressures are reached impact excitation becomes more important compared with radiative excitation, is not, therefore, applicable to the Saha theory.

The other quantity mentioned by Compton as supporting the transfer of the theory of cumulative ionisation to astrophysical examples is the accelerating potential difference applied between the electrodes of the tube. "At high voltages and low gas pressures there is good opportunity for ionisation by direct single impacts," contrasted with "the case of purely temperature ionisation, in which there is no potential drop at all. Here, presumably, cumulative ionisation is practically the only type, and resonance radiation is an essential agent in the process. This concept has an interesting application to the case of ionisation in the solar chromosphere, recently discussed by Saha . . . " (*loc. cit.*, p. 297). This leads to his and Russell's explanation of the Ba anomaly in the case of the sun.

Now the effective meaning of "potential drop" in collision ionisation is simply the kinetic energy which the applied potential gives to electrons, and this kinetic energy is as effective for ionisation whether it comes from the field set up between the electrodes of a tube by an external battery or whether it is due to the random velocities of thermal agitation in a gas. The exact correlation of King's furnace excitation of spectral stages with the excitation of the same stages by potentials applied to a thermionic tube, leaves no room for doubt on this. It seems, therefore, misleading to speak of purely temperature ionisation as a case of "no potential drop at all," and, hence, contrasted with the case of high voltages. The only differences between the cases of high potentials applied between electrodes in a gas and temperatures high enough for thermal ionisation are as follows:—

(1) Doppler effects due to electronic velocities in an externally applied field will be shifts of spectral lines, but Doppler effects due to the velocities of thermal agitation will be a broadening of lines, since there is no one resultant direction.

(2) In the imposition of motion by an external attraction, the ionising effectiveness of an electron will not only be proportional directly to the

gas pressure, but will involve also a function of the inverse pressure; since at high concentrations of atoms the smaller free path will not allow ionising velocity to be reached (Townsend, *Ionisation by Collision*, p. 10). In the random motion of thermal agitation in a gas there is no external attraction in any direction, and it does not matter in the collision what the free path has been.

Neither of these two only possible differences between thermal velocities and the velocities due to an applied field seem to justify the remarks quoted above on temperature ionisation; on potential as well as pressure considerations, therefore, the application of cumulative ionisation to a stellar atmosphere cannot be accepted on Compton's grounds.

4. Russell has used the idea of the disturbance of the thermodynamic equilibrium of the reversing layer by photospheric radiation to explain the Ba anomaly. This idea was first put forward by Compton in connection with his theory of cumulative ionisation. The points just raised against his method of applying that theory to a stellar atmosphere do not alter the possible importance of such a disturbance of equilibrium, provided the pressure and potential relations are emended for quantitative purposes. The success of cumulative ionisation theories to explain astrophysical anomalies must therefore be compared with that of alternative suggestions.

Now the original Saha theory could give no reason why Ba of ionisation potential 5·2 volts should be more completely ionised in the solar reversing layer and spots than Na of ionisation potential 5·1 volts, since the equilibrium between ionised and neutral atoms and electrons should at the same temperature and gas pressure depend solely on the energy required to dissociate them. Three explanations involving this and other apparent anomalies have so far been offered by Russell, by Saha himself, and by Plaskett:—

(i) Russell (*Astroph. Journal*, 1922 May), arguing from Compton's emphasis on the importance of radiative excitation, points out that the radiation from the hotter photosphere might result in more ionisation in the reversing layer than the strict equilibrium value, the increase being greater for Ba than for Na for the following two reasons: (*a*) Once ionised Ba atoms will be likely to be further excited by the appropriate frequencies in the continuous photospheric spectrum; Russell further suggests that the Ba atom excited towards second ionisation will be less likely to recombine with a free electron than would an atom whose second valency electron was in the normal state, and more liable to further ionisation. For Na, on the other hand, the correspondingly effective spark frequencies are in the ultra-violet where the continuous spectrum is weak. This he generalises as follows:—"An element which has strong enhanced lines in the region where the continuous spectrum is strong, will behave as if it was relatively easier to ionise than one which does not possess such lines" (*loc. cit.*, p. 358). (*b*) The D lines, which are the principal lines of the neutral Na atom, are stronger than the principal neutral lines of Ba in the Fraunhofer spectrum; neutral Na in the outer atmosphere of the sun is therefore more shielded from that part of the photospheric radiation which would excite it than is Ba.

Now Russell's argument referred to as (*a*) gives a valid reason why the alkaline earths when once ionised would be more easy to ionise a second time than the alkalies, provided that the mechanism is largely radiative, since the alkali spark frequencies are mostly in the ultra-violet. In the sun, however, we are only concerned with the spectra of once ionised atoms: the anomaly is the observed fact that Ba in the solar reversing layer is in a condition to absorb spark-lines while Na is not. The comparison between Ba^{++} and Na^{++} which would confirm the argument is not available. The further suggestion as to the difficulty of recombination to an atom with its second valency electron excited, deals with a region in which theories of atomic structure are largely speculative, and is not satisfactory if alternative explanation is available.

To explain the unequal primary ionisation, then, the weight of the argument must fall on (*b*). Now, here again, the contention is valid that the existence of D absorption in the reversing layer must shield neutral chromospheric Na from radiation that it might absorb and thus become excited. It is, however, with the Fraunhofer spectrum that the anomalous comparison with Ba appears; and here the Na atoms are absorbing strongly, as shown by the D lines, the very frequencies whose reduced intensity in the chromosphere Russell invokes to explain lack of excitation. It is not possible to use the actual fact that a frequency is being absorbed to explain why it cannot be absorbed unless different layers in the sun are concerned; the argument would only be valid if high chromospheric Na were discussed. Here, however, it would itself, if valid, be turned against the argument (*a*), the absorption of Ba spark lines producing the same effect that Russell claims for the neutral Na lines in (*b*). The two arguments are not, then, a satisfactory explanation of unequal primary ionisation in the reversing layer, and if made valid by considering different layers the second argument will vitiate the first.

(ii) Saha himself comments on the anomaly of Ba and Na (*Phil. Mag.*, 1922 December), and offers an alternative explanation. He suggests that "elements belonging to higher groups (in the periodic classification) are more easily ionised than elements in the preceding group, and the successive steps of ionisation follow each other in rather quick succession" (p. 1130). He explains this as due to a quantity called the "steric factor," which defines the symmetry of an atomic model and the position of electronic orbits; this factor effectively reduces the ionisation potential in the equilibrium equation by an amount increasing with temperature, and depending on the number of valency electrons the atom has in its outer ring. These electrons, he suggests, increase ionisation probability in two ways:

(*a*) If twice as many electrons are in the outer ring of element A as B there is twice as great a probability of one being lost by collision from A as from B by a given energy of the medium.

(*b*) Similarly, the greater number of valency electrons will partially shield A from electron capture, decreasing recombination probability in the case of elements high in the periodic table.

The principal difficulty in Saha's theory of the steric factor is that it applies to all forms of ionisation equally due to radiation, thermal

energy, and applied field energy; it should, therefore, lead to all experimental values of ionisation potentials being lower than the values deduced from convergence frequencies of spectra in the case of most elements. The tabulations of data (*e.g.* Foote and Mohler, ch. 3) show no such discrepancies that could be interpreted as more than naturally distributed errors.

Apart from this general consideration, and apart from doubtful points of atomic structure, such as the unknown position of orbits and their interpenetration, certain particular astrophysical examples seem to act in the opposite direction to that required by the theory. There are not many pairs of elements with similar ionisation potentials and different steric factors available for comparison: the first is, perhaps, the pair C and Mg. Saha gives the steric factor as large for C and small for Mg. The ionisation potential of Mg is 7·65 volts; for C it is quoted by Professor Fowler (*Proc. Roy. Soc.*, A, 1924 March) as from 7 to 8 volts, on a suggestion due to Saunders. On the theory of the steric factor, C^+ should appear at a lower temperature than Mg^+ in a stellar sequence; but whereas Mg^+ appears in Go stars, C^+ does not appear till B6 or B8. Again, N, an element with a very high steric factor on Saha's theory, persists in absorption lines of the neutral atom as far as stars of class Oe5, with a maximum intensity at B2, C^+ having a maximum at the same place in stellar sequence and beginning and persisting similarly. Now the ionisation potential of N is variously estimated at from 17 to 30 volts, in any case less than the second ionisation potential of C, which is given by Professor Fowler (*loc. cit.*) as $24{\cdot}3 + 7$ or 8 volts. But on Saha's theory the steric factor reducing the effective ionisation potential for N is greater than for C, *i.e.* N should be more easily ionised even if the ionisation potentials were the same.*

(iii) Plaskett (*Publ. Dom. Astroph. Obs.*, 1922) explains the greater ionisation of Ba than Na in the sun by the assumption that terrestrial relative abundance of elements is a safe guide to their relative abundance in the stars. The greater abundance of Na then requires the point in stellar sequence at which the D lines disappear to be postponed until class B8, while the disappearance of neutral Ba should occur at solar temperature.

Now Plaskett's theory explains satisfactorily, not only the persistence of neutral Na in stars as hot as the sun, but also the persistence of neutral H in a condition to absorb Balmer lines in O-type stars from which Mg^+ and He have already disappeared, since H and Na are known to provide 15 per cent. and 2 per cent. respectively of the number of atoms in the first ten miles of the earth's crust. Now the view that abundance of an element may affect the disappearance of its arc lines in the stellar sequence is not vitiated by Milne's criticism (*Observatory*, 1923 April) which has shown that Plaskett's treatment of recombination in his equations leads to a disagreement with known facts. Nevertheless, there are difficulties in the way of using the terrestrial abundance measurements to account for facts in star spectra; Plaskett has himself

* The place in stellar sequence of some lines important for these considerations is well summarised in diagrams, see Milne, *Proc. Phys. Soc.*, 1924 February, p. 107; Plaskett, *loc. cit.*, p. 356.

demanded for O-type stars an abundance of He 10^6 times its abundance on earth, and until more is known of chemical evolution the direct and the inverse uses of his theory are not equally valid; that is to say, the deduction of abundance from otherwise proved physical data in stars is possible, but the inverse method of assuming the abundance, as in his explanation of the Ba anomaly, is not generally reliable. For instance, Si and O with atomic abundance percentages on the earth of 16 and 55 respectively, either are not of the same stellar abundance or else do not bear out the spectroscopic effect of Plaskett's theory. N has a terrestrial atomic abundance of ·038 on Plaskett's scale of percentages, and hence its persistence is against Plaskett's theory as well as that of Saha. The absence of Li from the sun and its faintness in spots, unexplainable on Russell's theory owing to the absence of known enhanced lines, is also unexplained by Plaskett's theory, as it has an abundance greater than Ba though less than Na.

5. Consider, finally, how far cumulative ionisation in an atmosphere of disturbed thermodynamic equilibrium can be freed from the objections to Russell's and Compton's theories, and used to explain the anomalies mentioned. Using the results of 2, three conclusions emerge:—

Firstly, the fact that abundance is a factor of importance but that abundances are not safely calculable from terrestrial sources makes it difficult to claim any theory of any anomaly as the sole explanation.

Secondly, it was seen that the disturbance of the equilibrium by radiation alone will have a secondary effect on the efficacy of collisions also. This shows that cumulative ionisation of all types will be possible independently of particular mechanisms. Deductions can thus be drawn from the positions of energy levels apart from the method of transition from one level to the other suggested by Compton and Russell. Now the alkalies have only one distinct resonance potential while the alkaline earths have two: this will account qualitatively for the easier ionisation of the latter if ionisation potentials were equal, under the conditions of the atmosphere discussed. In the particular case of Ba and Na the latter requires energy for transition between levels corresponding to 2·09 and 5·11 volts, the former 1·56, 2·23, and 5·19 volts.

Thirdly, "τ" has been found greater for once ionised than for neutral atoms. This would be a reason for the rapidity of successive ionisations which Saha suggested as probably due to steric factor.

It seems that until values of "τ" and the critical potentials of higher elements are determined, these are the only considerations which are certain and effective and free from objection. Cumulative ionisation on these lines does point in the direction of the observed facts.

I am very grateful for the great kindness of Mr. E. A. Milne and Mr. F. J. M. Stratton in advice and criticism of the views underlying this note.

Physics Department:
Birmingham University.

Note on the Absorption of Radiation within a Star.

By S. Rosseland.

(*Communicated by Prof. A. S. Eddington, F.R.S.*)

1. In a series of papers Eddington * has discussed the theory of absorption and emission of radiation within a star in relation to his theory of the hydrostatic equilibrium of a gaseous star. On this latter theory the constitution of a star will depend intimately on the absorption coefficient of the material, and conversely it is possible to derive from stellar data a value of this absorption coefficient, the value being appropriate for the far interior of the star. In the papers referred to above Eddington is concerned with the calculation of what may be called the arithmetic mean value of the absorption coefficient, which is defined in such a way that if x_ν is the absorption coefficient for radiation of frequency, ν and $B(\nu)d\nu$ the energy of black body radiation in the interval ν to $\nu + d\nu$, the mean absorption coefficient x is given by

$$x = \frac{\int_0^\infty B(\nu) x_\nu d\nu}{\int_0^\infty B(\nu) d\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It will, however, appear that this coefficient only corresponds to the mean absorption coefficient entering into the stellar problem when x_ν is independent of the frequency, and for an exact treatment of the problem the averaging process has to be made in a different manner.

To see more clearly how things are brought about, consider briefly the manner in which the absorption coefficient enters into the stellar problem.† We assume the star to be spherically symmetrical. The flux of radiation must then at any point be symmetrical with respect to the direction to the centre of the star. Let

$$I_\nu \, . \, d\nu \, . \, d\sigma \, . \, d\omega$$

be the radiation in the frequency interval $d\nu$ passing per second through the surface element $d\sigma$ (placed normally to its direction) and contained in the solid angle $d\omega$. If θ is the angle formed by I_ν with the outward direction of the radius vector to the point in question, I_ν must be a function of θ, r and ν, and $d\omega$ can be taken equal to $2\pi \sin \theta d\theta$. Denoting the net outward flux of radiation per unit area with $F_\nu d\nu$, we obtain

$$F_\nu = 2\pi \int_{-\pi}^{+\pi} I_\nu \cos\theta \sin\theta d\theta \quad . \quad . \quad . \quad . \quad (2)$$

On the other hand, the energy density of radiation is

$$A_\nu = \frac{2\pi}{c} \int_{-\pi}^{+\pi} I_\nu \sin\theta d\theta \quad . \quad . \quad . \quad . \quad (3)$$

* *Monthly Notices*, **83**, 32, 98, 431 ; **84**, 104.

† *Cf.* A. S. Eddington, *Monthly Notices*, 77, 16, 596.

The fundamental equation of radiative transfer, the state being independent of the time, is

$$\frac{\partial I_\nu}{\partial s} = x_\nu \rho \left(\frac{cB(\nu)}{4\pi} - I_\nu \right) \quad . \quad . \quad . \quad . \quad (4)$$

where ∂s is an element of path, x_ν the absorption coefficient, and ρ the density, while $B(\nu)$ will in general be the energy density of black body radiation corresponding to the temperature of the medium at the point in question.* For the present case

$$\frac{d}{ds} = \cos\theta \frac{d}{dr} - \frac{\sin\theta}{r} \cdot \frac{d}{d\theta} \quad . \quad . \quad . \quad . \quad (5)$$

Multiplying (4) with $2\pi \cos\theta \sin\theta d\theta$, taking account of (5) and integrating from $\theta = -\pi$ to $\theta = \pi$, we obtain

$$F_\nu = -\frac{2\pi}{x_\nu \rho} \int_{-\pi}^{+\pi} \frac{\partial I_\nu}{\partial s} \cos\theta \sin\theta d\theta = -\frac{2\pi}{x_\nu \rho} \frac{\partial}{\partial r} \int_{-\pi}^{\pi} I_\nu \cos^2\theta \sin\theta d\theta + \frac{2\pi}{x_\nu \rho} \frac{1}{r} \int_{-\pi}^{\pi} \frac{\partial I_\nu}{\partial \theta} \cos\theta \sin^2\theta d\theta \quad (6)$$

When the asymmetry in the radiant field is so small that I_ν is approximately independent of θ, the first term on the right-hand side of (6) reduces to

$$-\frac{1}{3} \frac{c}{x_\nu \rho} \frac{\partial A_\nu}{\partial r}.$$

On the same assumption the term in $\frac{\partial I_\nu}{\partial \theta}$ must be small, and this will be the more the case on account of the small curvature of the isothermal surfaces. Neglecting this latter term we obtain the fundamental relation

$$F_\nu = -\frac{1}{3 x_\nu \rho} \frac{\partial A_\nu}{\partial r} \quad . \quad . \quad . \quad . \quad (7)$$

which gives the flux of radiation for each infinitely small frequency interval. It can be simply proved from (4) that the true energy density of radiation A_ν will differ from the energy of black body radiation $B(\nu)$ only by a term of the order of magnitude

$$-\frac{1}{x_\nu^2 \rho^2} \cdot \frac{\partial^2 A_\nu}{\partial r^2},$$

which for all cases relating to the interior of a star must be insignifi-

* If the material medium emits electro-magnetic radiation spontaneously, or at a rate which is not covered by the term giving the temperature radiation, an additional term must be added to the right-hand side of (4) to account for this emission. *Cf.* E. A. Milne, *Cambr. Phil. Soc. Proc.*, **21**, 701.

cantly small compared to A_ν itself. The total flux of radiation for all frequencies is therefore

$$F = -\frac{c}{\rho}\int_0^\infty \frac{\partial B(\nu)}{\partial r}\frac{d\nu}{x_\nu} \quad . \quad . \quad . \quad . \quad (8)$$

In his theory Eddington worked *ab initio* with the mean value of the absorption coefficient x, so that the total flux took the form

$$F = -\frac{c}{x\rho}\int_0^\infty \frac{\partial B(\nu)}{\partial r}\,d\nu \quad . \quad . \quad . \quad . \quad (9)$$

and x is thus defined by

$$\frac{1}{x} = \frac{\int_0^\infty \frac{\partial B(\nu)}{\partial T}\frac{d\nu}{x_\nu}}{\int_0^\infty \frac{\partial B}{\partial T}\,d\nu} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where we have differentiated with respect to the temperature instead of the radius r. From the above expression it appears that it is not properly the absorption coefficient itself which has to be averaged but its inverse value, the generating function being the derivative of the energy density with respect to temperature.

2. It is seen that if x_ν varies rapidly with ν, the two averaging processes (1) and (10) may give essentially different results. From the form of the expression (10) it is already possible to say something in general about the kind of absorption which will be determinative for stellar constitution. It will thus be seen that all absorption of a selective nature, as absorption in a true line spectrum, may be dismissed as contributing little to the final result, which will preponderantly be determined by the smallest values of the absorption coefficient occurring in the spectral interval where $\frac{\partial B(\nu)}{\partial T}$ has appreciable values. Consider next the case of absorption independent of frequency with absorption coefficient σ_1, on which is superposed strong absorption setting in at a certain frequency ν_m (absorption coefficient σ_2) and continuing indefinitely towards larger frequencies. The corresponding value of the stellar absorption coefficient as calculated by (10) will be of the order of magnitude σ_1 regardless of the value of σ_2, provided ν_m is situated sufficiently far to the short wave-length region for the integral $\int_{\nu_m}^\infty \frac{\partial B(\nu)}{\partial T}d\nu$ to be small compared to $\int_0^\infty \frac{\partial B(\nu)}{\partial T}d\nu$. This conclusion has a bearing on the absorption connected with ejection of electrons from the normal state of the atoms. According to the theory of ionisation * the fre-

* *Cf.* A. S. Eddington, *Monthly Notices*, 84, 104.

quency of the radiation to be absorbed for ejection of electrons to be possible will be of the order $\nu_m = 7\frac{k\mathrm{T}}{h}$, and this value is not much dependent on the precise value of the atomic number provided it is large enough for a large majority of the atoms to retain at least one electron. When the atomic number is so large that L- and M-electrons are also retained, the critical value of ν may drop a little below the above value, but this is immaterial for the main conclusion. The maximum value of $\frac{\partial \mathrm{B}(\nu)}{\partial \mathrm{T}}$ occurs for about $\nu = 1{\cdot}7\frac{k\mathrm{T}}{h}$, and at $\nu = 7\frac{k\mathrm{T}}{h}$ the ordinate has dropped to an insignificant fraction of its maximum value. It seems, therefore, that the corresponding absorption process cannot contribute much to the stellar absorption coefficient. On the other hand, the ejection of electrons from atoms in excited states may take place for frequencies for which $\frac{\partial \mathrm{B}(\nu)}{\partial \mathrm{T}}$ is large, and the same is the case for absorption by free electrons.

It will be seen that the discrepancy between theory and observation found by Eddington * will not be removed simply by averaging according to (10) instead of (1); on the contrary, the discrepancy will be expected to come out essentially larger. The closer discussion of the theory of the absorption coefficient itself and of the possibilities for overcoming the difficulties mentioned above will be treated separately in a second paper.

Copenhagen,
Institute of Theoretical Physics:
May 5.

A Comparison of Characteristic Features of Related Pairs of Sunspots in Two Successive Cycles. By Professor H. F. Newall, F.R.S. and Mrs. Beech.

Hale's discovery of the reversal of relative magnetic polarities in bipolar groups of sunspots suggests the idea that it might be possible to detect differences in characteristics, other than magnetic, of related pairs of sunspots in two successive cycles. The present note gives the results of an attempt which aims at comparing the relative movements and relative areas of pairs of spots, and is based on a study of the data of the Greenwich photoheliographic results.

Certain groups of spots were picked out as satisfying the conditions: (1) that each should contain only two spots marked in the Greenwich record with the identification letters *a* and *b*; (2) that these pairs of lettered spots should appear on at least four consecutive days; (3) that only such pairs were to be utilised as had broken out in sight not further than 70° longitude east of the central meridian; and (4) that they must be first appearances, not later appearances of recurrent or

* *Monthly Notices*, **84**, 104.

intermittent groups. Out of a first selection of 195 groups, 102 were rejected as failing to satisfy one or another of the four conditions, and the remaining 93 have been examined. They are distributed as follows:

	Cycle III., 1901–1912.	Cycle IV., 1913–1920.
Northern hemisphere	25	16
Southern hemisphere	32	20
	57	36

The heliographic positions of each pair were plotted on squared paper, so that the relative movements of the components could be seen at a glance. As is well known, the relative movement in longitude is most conspicuous. The pairs were then classified for each hemisphere in each cycle into categories according to the directions of the movements in longitude of the leading and following spots. The categories are indicated in the top line of the table by arrows or by the letter S: an arrow ← meaning movement of a spot to smaller longitude; S meaning stationary. Thus S → means that the following spot is stationary and the leading spot is moving forward in longitude. Table I. shows that in that category in the northern hemisphere of Cycle III. there were 11 pairs, the mean separation being 4°·1 at the beginning of the observed movement and 7°·5 at the end. Thus the mean range of movement in this category was 3°·4. The actual separations of individual pairs are so similar that it does not seem necessary to give more than the mean values.

It was found that among the 93 available pairs there was not a single case in which the separation of a pair of spots diminished. It may be recalled that Carrington in his *Memoir on Sunspots* (p. 245) noted 33 cases of divergence; but he also noted 11 cases in which that effect was absent, *i.e.* 25 per cent. Table I. shows that 17 out of 93, or 18 per cent., were relatively stationary pairs.

TABLE I.

Table of Mean Initial and Mean Final Separations of Following and Leading Spots.

Categories.	← ←.	S S.	→ →.	← S.	S →.	← →.	All.
Cycle III.							
Northern spots	7° 8°	5°·2 5°·2	5°·5 8°·0	3° 6°	4°·1 7°·5	4°·2 8°·2	4°·6 7°·1
	(1) 1	(6) 0	(2) 2·5	(1) 3	(11) 3·4	(4) 4·0	(25) 2·5±0·26
Southern spots	...	3·4 3·4	4·0 5·2	3 7	3·5 7·4	2·5 8·7	3·4 6·4
	(0)	(7) 0	(4) 1·2	(1) 4	(16) 3·9	(4) 6·2	(32) 3·0±0·25
Both hemispheres	7 8	4·2 4·2	4·5 6·2	3 6·5	3·7 7·5	3·4 8·5	3·9 6·7
	(1) 1	(13) 0	(6) 1·7	(2) 3·5	(27) 3·8	(8) 5·1	(57) 2·8±0·17
Cycle IV.							
Northern spots	...	2 2	4 8	...	3·3 9·6	4·8 9·7	4·0 9·0
	(0)	(1) 0	(2) 4	(0)	(6) 6·3	(7) 4·9	(16) 5·0±0·36
Southern spots	5·5 6·0	3·3 3·3	...	...	2·7 9·3	3·4 8·5	3·3 7·7
	(2) 0·5	(3) 0	(0)	(0)	(7) 6·6	(8) 5·1	(20) 4·4±0·53
Both hemispheres	5·5 6·0	3·0 3·0	4 8	...	3·0 9·5	4·1 9·1	3·6 8·3
	(2) 0·5	(4) 0	(2) 4	(0)	(13) 6·5	(15) 5·0	(36) 4·7±0·33

The numbers of available pairs are too small to justify the laying of much weight on the distribution among the various categories. If the mean values (last column) be taken without regard to the categories, it would appear that the divergence of spots [mean values printed in clarendon in Table I.] is greater in Cycle IV. than in Cycle III. in each hemisphere and, consequently, also in the two hemispheres combined. But this can only have any significance if the duration of visibility of the components is the same in the two cycles.

Accordingly the duration of visibility of the leading and following spots was tabulated for every pair, and the mean values in days are recorded in Table II.

TABLE II.

	Cycle III.			Cycle IV.		
	No. of Pairs.	Mean Duration Days.		No. of Pairs.	Mean Duration Days.	
		Followers.	Leaders.		Followers.	Leaders.
Northern	(25)	5·0	5·0	(16)	5·8	6·8
Southern	(32)	4·7	5·1	(20)	5·8	6·1
All	(57)	4·8	5·1	(36)	5·8	6·4
		5·0			6·2	

If we utilise the mean values of the separations from Table I. and the mean values of the durations from Table II. to find the mean daily divergence for the two cycles, we find

0°·56 per day for Cycle III., 0°·75 per day for Cycle IV.

The characteristic feature generally recognised in regard to the relative areas of the components of pairs of spots is that the leading spot is, in general, more stable than the following and has the larger area. It is, moreover, generally formed earlier than the following spot. The relative areas of the leaders (L) and followers (F) were tabulated, and the results are summarised in Table III., in which the numbers indicate the frequency of occurrence of cases in which (for instance) the area of the leader exceeds that of the follower both at the beginning of the observations of the pair and at the end. Thus, in Cycle III., northern hemisphere, there were 11 pairs in which the leader had the greater area both at beginning and end (LL); there were 3 pairs in which the follower had a greater area at the beginning but a smaller area at the end (FL), and so on.

TABLE III.

	Cycle III.					Cycle IV.				
	No. of Pairs.	LL.	FL.	LF.	FF.	No. of Pairs.	LL.	FL.	LF.	FF.
Northern	(25)	11	3	8	3	(16)	7	6	2	1
Southern	(32)	14	5	8	5	(20)	10	3	4	3
All	(57)	25	8	16	8	(36)	17	9	6	4
Percentages		44	14	28	14		47	25	16	11

Conclusion.—The evidence appears at first sight to indicate slight systematic differences between the two successive cycles in regard to relative movements. It is given for what it is worth; but it would seem to be so slight that it should be regarded only as justification for examination of more material before a definite conclusion can be safely drawn.

1924 *May* 9.

The Magnetic Disturbance 1924 *January* 29–30.

By Rev. A. L. Cortie, S.J., D.Sc., F.Inst.P.

The magnetic disturbance of 1924 January 29–30 was not only of marked activity in itself, but was notable from the circumstance of its occurrence at a period of great solar quietude. The greatest phase of the storm took place between the times $29^d\ 17^h\ 52^m$ and $30^d\ 3^h\ 0^m$ (civil time). Rapid oscillations of the needles characterised this interval. The extreme ranges were 36′·0 in declination, 169γ in horizontal force, and 292γ approximately in vertical force, where $1\gamma \equiv 10^{-5}$ C.G.S. unit. We may compare with these the mean ranges of the needles on the five quiet days of 1924 January, which were 2′·6 in declination, 10·6γ in horizontal force, while the vertical force magnet was practically stationary. Such a great range in vertical force would be noticeable in any magnetic storm. The magnetic disturbance was also accompanied by an auroral display. It had been preceded by a disturbance of moderate character at a 27-day interval approximately, on January 3, the ranges being 14′·0 in declination and 70γ in horizontal force.

The quiet state of the sun's surface is set forth in the following table, derived from the Stonyhurst records, the areas of the spots being expressed in 1/5000 of the visible disc.

	Mean Area Spots.	Number of Spotless Days Observed.	Percentage of Spotless Days.
1923 Nov.	0·80	7	33
Dec.	0·20	9	69
1924 Jan.	0·00	18	100
Feb.	0·60	11	65
Mar.	0·15	21	84
Apr.	0·96	12	48

The quietest of the last six months was 1924 January, when the sun was entirely free from spots on the 18 well-distributed days on which it was observed, and when also such faculæ as were seen were of a faint and diffused nature. Here then is an instance of a magnetic storm of considerable activity occurring without any accompanying visible phenomenon on the sun. But we shall show that it was not an isolated or independent magnetic disturbance, but was a member of an intermittent series that was connected with solar disturbance.

We have already noticed that it was preceded at an approximate 27-day interval by a moderate magnetic disturbance on 1924 January 3.

By reckoning backwards at intervals of between 26 and 27 days we arrive at the date 1923 October 14, which was the first of five successive days of considerable magnetic disturbance, occurring too between periods of magnetic calm. The magnetic disturbances 1923 October 14–18 began with a sudden movement of the needles on October $14^d\ 21^h\ 18^m$. The maximum ranges were 35′ in D, 153γ in H, and 180γ in V. In the storm of 1924 January 29–30, the time of commencement was $29^d\ 17^h\ 52^m$. The whole interval between these two times covered four synodic rotations of the sun, giving a mean period 26·71 days. This synodic period corresponds to a sidereal period of 24·89 days, and a daily rotation of the sun of 14°·46. The mean latitude corresponding to this rate of daily rotation derived from Carrington's and Spoerer's formulæ is ±4°·4.

With regard to the longitude, the mean longitude of the sun's central meridian on the two dates will furnish a clue. It works out at 241°·4. Consequently, if there was a solar disturbance connected with the intermittent series of magnetic disturbances, of which the great storm of 1924 January 29–30 was a member, it must be looked for in the neighbourhood of the position indicated by these co-ordinates. In the Stonyhurst lists of sun-spot disturbances there was a group, No. 184, observed 1923 October 9–20, the mean latitude of which was +3°·9, and the mean longitude of the chief spot of the group 223°·9. It was not a large spot, its maximum area having been 0·8. But the same position, allowing for drift in longitude, had been disturbed by a small group, observed 1923 July 27–29, with maximum area 0·2 units. Subsequently faint faculæ were recorded on 1923 December 13, in mean latitude +9°·0 and longitude 221°·0, and again on 1924 January 27–31, in mean latitude +5°·0 and longitude 240°·0. The result is :—

(*a*) From magnetic phenomena. Computed.

Mean synodic period 26·71 days.
Mean sidereal period 24·89 days.
Daily rotation of the sun in arc 14°·46.
Mean latitude ±4°·4.
Mean longitude of C.M. 241°·4.

(*b*) From solar spots and faculæ. Observed.

Mean latitude +6°·0.
Mean longitude 228°·3.

There were therefore signs of intermittent disturbance on a definite area of the sun's surface, which corresponded in its synodic rotations with intermittent magnetic activity. But there was no parity in intensity, nor correspondence in time, between the phases of solar and of terrestrial magnetic disturbance. The magnetic storm of 1924 January 29–30, though occurring at a time of apparent solar calm, was associated with a disturbed area of the sun's surface. This case is by no means unique or singular. Reference may be made to a summary account of such series of magnetic disturbances, published in *The Observatory*, **46**, No. 593.

Stonyhurst College Observatory :
1924 *May* 6.

On the Possible Rate of Increase of Mass of the Sun.

By Harold Jeffreys, M.A., D.Sc.

It can be shown that the part of the secular acceleration of the sun due to tidal friction must lie between $\frac{1}{13}$ and $\frac{1}{5}$ of the secular acceleration of the moon similarly caused; if the latter amounts to 9″/(century)², the secular acceleration of the sun should be between 1″·8 and 0″·7/(century)², the most probable value being about 1″·3. Now Fotheringham's determinations seem to indicate that the solar secular acceleration is probably about 3″·0/(century)², and that it cannot be less than 2″·1/(century)². It is therefore possible that a secular acceleration of the sun of the order of 1″·0/(century)² may have to be explained otherwise than by tidal friction.

If the mass of the sun is varying, the conditions given by Kepler's second and third laws are:

$$r^2\dot{\theta} = h = \text{const.}$$
$$r^3\dot{\theta}^2 = f\mathrm{M}$$

in a familiar notation; hence

$$\dot{\theta} = \frac{(f\mathrm{M})^2}{h^3},$$

and if the increase of $\dot{\theta}$ in a century is 1″/century, we have in a century

$$\frac{\delta\mathrm{M}}{\mathrm{M}} = \tfrac{1}{2}\frac{\delta\dot{\theta}}{\dot{\theta}} = \tfrac{1}{2}\frac{1''}{\text{century}} \div \frac{36000^\circ}{\text{century}}$$
$$= \frac{1}{2{\cdot}6 \times 10^8}.$$

Thus the sun's mass would have to increase by 1 part in $2{\cdot}6 \times 10^{10}$ per year to complete the dynamical explanation of the secular acceleration of the sun.

Small as this increase appears, it is too great to be admissible. In a former paper * I estimated the sweeping power of a gravitating planet moving through a Maxwellian distribution of particles moving at random, and the formula (19) on p. 99 of that paper is directly adaptable to the absorption of interstellar matter by the sun. If P be the distance to which a particle would approach if undeflected by solar gravity, the formula gives at once, provided the sun's velocity is not great compared with the average velocity of the matter,

$$[\mathrm{P}^2]_{\text{mean}} = O(4f\mathrm{M}a^2\mathrm{R})$$

where R is the sun's radius and a the "constant of precision" of the distribution, so that $a^2c^2 = \frac{3}{2}$ where c is the mean square velocity of the particles. If ρ be the density of interstellar matter and U the sun's velocity, the rate of absorption of the matter is of order $\rho\mathrm{U}.\pi.[\mathrm{P}^2]_{\text{mean}}$ or $6\pi f\mathrm{M}\mathrm{R}\rho\mathrm{U}/c^2$. If, then, this rate is equal to

* *M.N.R.A.S.*, **77**, 84–112, 1916. In the formula quoted e^{-x^2} is an error in copying for e^{x^2}.

$M/2{\cdot}6 \times 10^{10}$ per year, and $c = 6 \times 10^6$ cm./sec., $U = 2 \times 10^6$ cm./sec., we find

$$\rho = O(1{\cdot}5 \times 10^{-16}) \text{gm./cm.}^3.$$

Professor Eddington informs me that this density is much greater than is admissible on astrophysical grounds; but it is also inconsistent with observations within the solar system itself. The density of matter being absorbed by the sun would naturally increase towards the sun; thus the interstellar density must be less than that of the zodiacal matter, which I have already estimated * at 10^{-17} gm./cm.^{3} at the very most, from considerations based on its luminosity. Thus there cannot be as much interstellar matter as is needed to account for the secular acceleration of the sun.

On the Internal Constitution of Jupiter and Saturn.
By Harold Jeffreys, M.A., D.Sc.

The Radau-Darwin theory of the figure of the earth is capable of being applied to determine the moments of inertia of Jupiter and Saturn from their ellipticities, and hence to obtain an important datum concerning the internal constitution of each planet. The Radau approximation is

$$\frac{C}{Ma^2} = \tfrac{2}{3}\{1 - \tfrac{2}{5}\sqrt{(1+\eta_a)}\} \quad . \quad . \quad . \quad . \quad (1)$$

where C is the greatest moment of inertia of the planet, M its mass, a its radius, and

$$\eta_a = \frac{5}{2}\frac{m}{\epsilon} - 2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ϵ is the surface ellipticity and

$$m = \frac{a\omega^2}{g} = \frac{a^3\omega^2}{fM} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

ω being the angular velocity of rotation of the planet, f the constant of gravitation, and g surface gravity.

The proportional error in this approximation is partly due to the neglect of the square of the ellipticity and partly to the difference between

$$\psi(\eta) = \frac{1 + \frac{1}{2}\eta - \frac{1}{10}\eta^2}{\sqrt{(1+\eta)}} \quad . \quad . \quad . \quad . \quad (4)$$

and unity. When η lies between 0 and 1 the latter difference is less than 1 per cent.

The ellipticity ϵ can be found directly from observation, but its values for Jupiter and Saturn can be found more accurately from the

* *M.N.R.A.S.*, 80, 138–154, 1919.

perturbations of the satellites; to the first order the portions of the motions of the nodes and pericentres due to the oblateness of the primaries are proportional to $\epsilon - \frac{1}{2}m$. For Jupiter, Darwin quotes * from J. C. Adams the value

$$\epsilon - \tfrac{1}{2}m = 0{\cdot}0216$$

and

$$m = 0{\cdot}0815.$$

From these Darwin gets

$$\frac{5m}{2\epsilon} = 3{\cdot}2646,$$

and therefore

$$\eta_a = 1{\cdot}2646.$$

The Radau approximation therefore gives

$$\frac{C}{Ma^2} = 0{\cdot}265.$$

There will be an error possibly of a few per cent. arising from the neglect of the square of the ellipticity; $\psi(\eta) - 1$ is about 0·02.

For Saturn, H. Struve gives †

$$\tfrac{1}{2}m = 0{\cdot}08494$$
$$\epsilon = 0{\cdot}1031,$$

whence

$$\eta_a = 2{\cdot}12.$$

The Radau approximation gives

$$\frac{C}{Ma^2} = 0{\cdot}198.$$

In this case $\psi(\eta_a) - 1$ is about 0·09, but the proportional error is a fraction of this expression.

For a homogeneous planet

$$\frac{C}{Ma^2} = 0{\cdot}400.$$

For the earth this ratio is 0·333. It follows that Jupiter and Saturn must be much more condensed towards the centre than is the case for the earth. This result was inferred by Darwin in the paper cited above, on the supposition that Laplace's law of density holds in these planets; here it has been inferred on the supposition that the Radau approximation is valid, which assumes little more than that the density increases inwards and that there is no tangential stress within the planets.

The above results enable us to fix upper limits to the surface densities of the planets. The moment of inertia of a planet is greater

* *Sci. Papers*, **3**, 60.

† *Publ. d. l'Observatoire Central Nicolas, Pulkovo*, ser. 2, **11**, 1898, 233. I am much indebted to Dr. J. Woltjer for this reference.

than it would be if the whole volume had the density of the exterior and the mean density was maintained at its actual value by a particle of infinite mass at the centre. If ρ_a is the surface density and ρ the mean density, we have

$$\frac{C}{Ma^2} > 0{\cdot}400\frac{\rho_a}{\rho}.$$

The mean densities of Jupiter and Saturn being respectively 1·36 and 0·63, we see that for Jupiter the surface density must be less than 0·90, and that for Saturn it must be less than 0·31. These limits have been computed on a necessarily extreme hypothesis; the actual surface densities must be much less, and the densities throughout considerable fractions of the volumes must be less than these assigned limits.

With the aid of these results it is possible to make certain inferences with regard to the physical constitution of Jupiter and Saturn. The temperatures of the planets have been measured by Menzel * and found to be about 150° A. (−123° C.). The densities of some of the lightest known substances in the solid and liquid states are as follows, quoted from Landolt-Börnstein's tables :—

	Liquid.	Solid.
Hydrogen	0·070 − 0·075	0·076
Helium	0·122	0·188
Ammonia	0·69	
Methane	0·324	0·477
Nitrogen	0·45	1·02
Oxygen	1·13	1·425
Carbon monoxide	0·311	
Carbon dioxide	1·057	1·53 − 1·63
Ethane	0·466	

Of these densities the only ones less than the maximum surface density for Saturn are hydrogen and helium, whose boiling-points are −253° and −269° C. respectively. That of carbon monoxide is −190°, and that of methane −164°. It is therefore impossible to reconcile our data for Saturn with a wholly solid constitution; a large fraction of the volume of this planet must be gaseous.

In an earlier paper on a similar topic † I showed that the radiation of the outer planets, if they were as hot as 30° C. at the surface, would have been sufficient to remove all the initial heat they could reasonably be supposed to have possessed; I inferred that these planets were probably at nearly the temperatures maintained by the sun's radiation. For Jupiter the surface temperature should be about 120° A., and for Saturn about 90° A. Since that time Menzel has found by observation that the temperatures of these planets are about 150° A. Thus his results confirm mine in so far as they contradict the traditional view that these

* *Astrophysical Journal*, 1923 September.

† *M.N.R.A.S.*, 83, 350–354, 1923. On the first page of the paper the mass of Jupiter has been wrongly written as 2×10^{33} gms. instead of 2×10^{30} gms.: the calculation has been carried out with the correct value.

planets are very hot even near the surface; but they are higher than can apparently be explained by solar radiation alone, even with any variations in the radiative properties of the surfaces that seem admissible.

The discrepancy cannot be explained by original heat. If such heat came from the gaseous part of a planet, we should have a hot atmosphere cooling by radiation from the top and therefore thoroughly stirred by convection currents. It therefore could not take long to lose its original heat. An argument in my former paper is available to show the inadequacy of original heat conducted from the solid interior.

The possibility remains that the heat is new heat generated from radioactive sources, and radiated soon after it is generated. The intensity of solar radiation on Jupiter is 3×10^{-4} cal./cm.^{2} sec., and an equal or greater amount from radioactivity would be required to account for Menzel's temperature. Now uranium generates heat at the rate of $2{\cdot}3 \times 10^{-8}$ cal./gm. sec. Hence the heat evolved requires about $1{\cdot}3 \times 10^4$ gms. of uranium per square centimetre of the surface of Jupiter. This is a very large amount; the corresponding amount for the earth is about 10 gms. per square centimetre. Further, the uranium must be within some 300 kms. of the surface, otherwise heat conducted from it would not have had time to affect surface conditions appreciably, with any reasonable estimate of the thermal conductivity.

In my earlier paper I assumed without calculation that the satellites of the great planets would be unable to retain atmospheres. Such retention appears, however, to be possible if we adopt the new estimates of the temperatures. For JI, for instance, the mass is $\frac{1}{7\frac{1}{0}}$ of that of the earth, and the diameter 0·31 of that of the earth. If v be the velocity of escape from the surface, v^2 is proportional to M/a, and therefore is $\frac{1}{2\frac{1}{2}}$ of its value for the earth. Thus if JI was as hot as the earth, and the earth could retain hydrogen (H_2), as is probable, JI could retain carbon dioxide; but if the temperature of JI is 150° A. and that of the earth 280° A., JI could retain a gas of molecular weight 24. Thus the larger satellites may well have atmospheres. If so, the variation among their densities may be attributed to the different depths of their gaseous portions, the depths on the whole being greater, and the densities therefore smaller, for the larger satellites. Satellites and planets alike may well have solid parts with densities comparable with those of terrestrial rocks.

Let us, then, investigate the possibility of Jupiter and Saturn being composed each of an outer gaseous layer of radius a and inappreciable density, enclosing a layer of ice and solid carbon dioxide of radius $a\beta$ and density 1, which itself encloses a rocky core of radius $a\gamma$ and density 3. The density and moment of inertia for Jupiter give the equations

$$\beta^3 + 2\gamma^3 = 1{\cdot}36$$
$$\beta^5 + 2\gamma^5 = 0{\cdot}91.$$

These are satisfied by $\gamma/\beta = 0{\cdot}725$, $\beta = 0{\cdot}917$, $\gamma = 0{\cdot}663$. For Saturn similarly we have the equations

$$\beta^3 + 2\gamma^3 = 0{\cdot}63$$
$$\beta^5 + 2\gamma^5 = 0{\cdot}31.$$

These are satisfied by $\gamma/\beta = 0{\cdot}56$, $\beta = 0{\cdot}773$, $\gamma = 0{\cdot}433$.

The existence of these solutions affords some evidence for the truth of the hypothesis they are derived from; for though theoretically solutions must exist, there is no *a priori* reason why they should satisfy the physical relations $1>\beta>\gamma>0$. The densities of the four great satellites of Jupiter and Titan are consistent with these bodies being composed of rock and ice, with possibly some atmosphere; the lighter satellites of Saturn may be a white porous rock or even snow, as Mr. Hepburn has suggested.

To sum up, the available evidence regarding Jupiter and Saturn is consistent with the hypothesis that each is composed of a rocky core, surrounded by a thick layer of ice, which is covered by an atmosphere whose depth is in the one case about 9 per cent. and in the other 23 per cent. of the radius of the planet. The atmospheres probably consist mainly of hydrogen, nitrogen, oxygen, helium, and perhaps methane; they are cloudy, the clouds consisting of particles of some substance, perhaps solid carbon dioxide, which is probably gaseous in the earth's atmosphere. Both on theoretical and observational grounds, the effective temperatures as regards radiation must be very low. Menzel's observations, if confirmed, would imply a considerable amount of radioactive matter in the outer surfaces of the planets. The upward transference of heat might, in its turn, account for the atmospheric disturbances; but it would be premature to assert that these cannot be explained otherwise.

Observations of Nova Persei (1901), 1923–24.

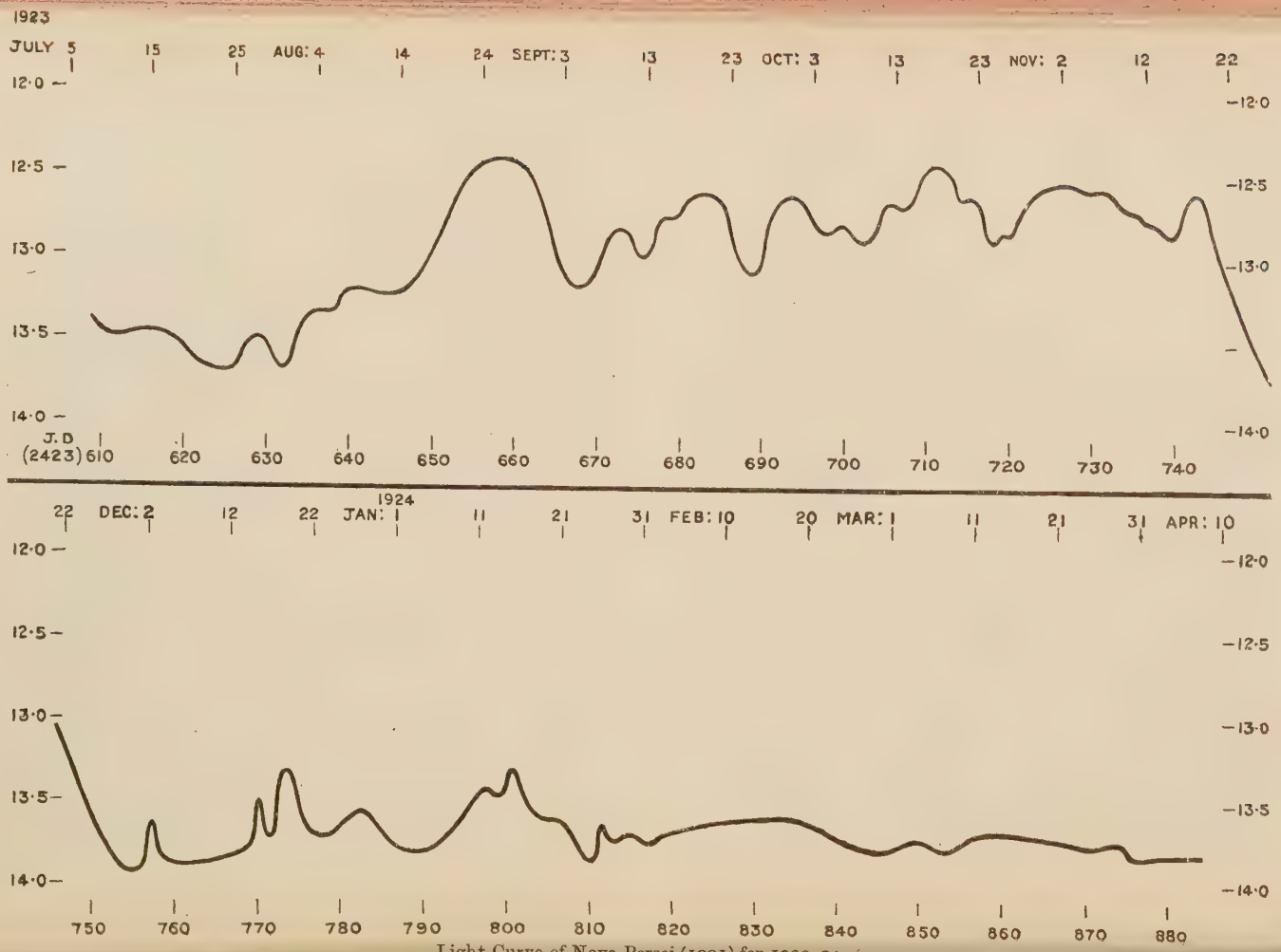
By W. H. Steavenson.

Photometric observations of this Nova were made on 123 nights between 1923 July 4 and 1924 April 8. Nearly all the estimates were made with the 6-inch Wray refractor, the comparison stars being the same as those used by the late Professor Barnard and myself in former years. I believe that the probable error of most of the observations does not exceed $0^m{\cdot}1$, though it may be as much as $0^m{\cdot}2$ in those which are indicated as approximate only. The estimates are as follows:—

Date.	Mag.	Date.	Mag.	Date.	Mag.
1923 July 4	<12·64	1923 July 28	13·5±	1923 Aug. 15	13·18
8	13·40	31	13·7±	18	12·96
11	13·5±	Aug. 2	13·44	21	12·64
13	<12·64	3	13·38	22	12·53
14	13·47	4	13·33	28	12·43
18	13·5±	5	13·32	30	12·53
20	13·62	6	13·33	31	12·64
21	13·67	7	13·22	Sept. 3	13·13
25	13·7±	8	13·20	5	13·18
26	13·6±	11	13·2±	6	13·13

Date.	Mag.	Date.	Mag.	Date.	Mag.
1923 Sept. 8	12·91	1923 Oct. 27	12·83	1924 Jan. 3	13·81
9	12·83	28	12·69	4	13·81
11	12·90	Nov. 2	12·53	6	13·78
12	13·00	3	12·53	12	13·43
13	13·0±	4	12·56	14	13·48
15	12·74	7	12·59	15	13·30
16	12·74	10	12·69	19	13·6±
17	12·69	11	12·71	20	13·6±
18	12·64	12	12·74	25	13·86
21	12·64	14	12·81	26	13·63
22	12·74	15	12·84	27	13·72
23	12·91	16	12·82	29	13·7±
26	13·1±	17	12·71	31	13·70
27	12·90	18	12·59	Feb. 1	13·76
28	12·74	19	12·64	2	13·71
30	12·61	22	13·13	14	<12·64
Oct. 4	12·83	30	13·91	16	13·6±
6	12·79	Dec. 2	13·9±	28	13·8±
9	12·9±	3	13·63	Mar. 4	13·73
12	12·66	4	13·83	5	13·73
13	12·66	9	13·86	8	13·8±
14	12·69	15	13·77	9	13·75
15	12·62	16	13·48	12	13·7±
16	12·53	17	13·73	16	13·7±
17	12·43	18	13·42	24	13·75
20	12·53	19	13·3±	29	13·75
21	12·64	20	13·4±	30	13·82
22	12·62	23	13·7±	Apr. 1	13·82
23	12·65	24	13·7±	5	<13·5
25	12·91	28	13·56	6	<13·5
26	12·83	1924 Jan. 2	13·81	8	13·8±

On drawing a curve through these observations it will be seen that between the middle of August and the middle of November the Nova was, on the whole, above its average brightness, and that during this time it underwent a series of oscillations of a somewhat regular character, the period averaging about 15 days. After November 19 there was a rapid and considerable fall to nearly 14th magnitude, and thereafter the Nova remained well below its average brightness for the remainder of the apparition. After the fall in November there were some rapid and apparently irregular oscillations, but the amplitude of these gradually diminished, and from



Light Curve of Nova Persei (1901) for 1923–24.

the end of February the magnitude did not vary as much as $0^{m}\cdot1$ in either direction.

The curve for 1923–24 is less similar to those for 1921–22 and 1922–23 (*M.N.*, 83, 397) than these latter are to one another. The faintness of the Nova at the beginning and end of the periods covered, and its relative brightness in late summer and autumn in each case, are the only points of resemblance.

As in previous years, the Nova has been seen, on the clearest and darkest nights, to be closely involved in nebulosity, but the actual image appears to be stellar.

Photometric Observations of Nova Persei 1901, 1923 *August to* 1924 *April, made at Gorse Cliff School, Boscombe.* By B. M. Peek.

In the following table are given the results of photometric observations of Nova Persei 1901 made on 117 nights during the season 1923–24, when the Nova was suitably placed for observation.

Whenever possible the $12\frac{1}{4}$-inch reflector was employed, but on one or two occasions the 5-inch refractor or the instruments of friends were used.

Table of Individual Observations.

Date.	Mag.	Date.	Mag.	Date.	Mag.
1923		1923		1923	
Aug. 21·46	12·64	Oct. 6·50	12·84	Nov. 4·35	12·53
22·58	12·59	9·50	12·98±	6·42	12·64
24·63	12·44	12·38	12·67	7·29	12·59
26·58	12·3±	13·42	12·69	9·42	12·80
Sept. 5·54	13·18	14·38	12·52	10·38	12·74
7·46	13·00	15·46	12·44±	11·42	12·86
8·42	12·98	16·39	12·12 *	12·44	13·00
9·38	12·94	16·48	12·48 *	14·32	12·80
10·42	13·00	17·42	12·35	15·35	12·97
11·46	13·04	20·42	12·25±	16·46	12·88
12·42	13·12	21·38	12·43	17·35	12·69
15·42	12·90	22·38	12·40	18·40	12·82
16·42	12·85	23·38	12·48	19·48	12·80
18·42	12·73	24·42	12·75±	21·46	<12·64
21·42	12·7±	25·38	12·85±	23·50	<12·64
22·42	12·77±	26·29	12·85±	24·42	<12·9
23·46	13·07	27·38	12·95±	25·46	<13·3
26·46	13·0±	28·38	12·67	26·29	13·63
27·46	13·01	31·38	12·80	30·35	13·82
29·50	12·51	Nov. 1·35	12·67	Dec. 2·49	13·90
Oct. 3·50	12·59	2·31	12·75	3·29	13·56
4·46	12·69	3·39	12·62±	4·27	13·74

* The estimate on Oct. 16·39 probably makes the star too bright. There is, however, little doubt that a rapid change occurred between this time and Oct. 16·48. The record contains the entry $9\frac{1}{4}$ hours certainly $>12\cdot32$; $11\frac{1}{2}$ hours certainly $<12\cdot32$.

Table of Individual Observations—continued.

Date.	Mag.	Date.	Mag.	Date.	Mag.
1923		1924		1924	
Dec. 5·38	13·72±	Jan. 6·27	13·70	Feb. 27·49	<13·5
6·35	13·63	9·30	13·84±	28·34	13·82±
8·44	13·85±	14·42	<13·1	29·31	13·77
9·26	13·65	25·33	13·65	Mar. 2·33	13·82
10·43	13·50	27·29	13·76	3·30	13·77±
13·32	13·75	29·34	13·80±	4·32	13·75
14·23	13·63±	31·39	13·77	5·44	<13·5
15·39	13·79±	Feb. 1·33	13·82	6·31	13·8±
16·49	13·71	2·34	13·84	7·32	13·80
17·44	13·77±	3·32	13·79	8·38	13·82
18·38	13·42	4·38	13·74	9·33	13·82
19·33	13·39±	6·38	13·77	10·34	13·77
20·31	13·4±	8·30	13·62	11·33	<13·47
23·31	13·65±	9·30	<13·5	12·34	<13·67
24·38	13·65±	22·31	13·7±	13·31	<13·47
28·25	13·67	23·29	13·75±	24·35	<13·2
30·27	13·77±	24·36	13·77	Apr. 1·33	13·80
		26·30	13·75		

It has often been noticed that lack of transparency or the presence of mist or cloud seems to reduce the apparent brightness of the nova by about 0·1 mag. more than that of the comparison stars. This is consistent with the existence of luminous nebulosity around the star-disc.

Occultation of α Tauri 1924 *April* 8, *observed at Gorse Cliff School, Boscombe.* By B. M. Peek.

Longitude 1° 50′ 41″·7 W. Altitude
Latitude 50 43 22·4 N. 120 ft.

Instruments, 12¼-in. reflector, 5-in. refractor.

Times—	D.	R.
	$8^h\ 46^m\ 20^s{\cdot}5$	$9^h\ 34^m\ 49^s{\cdot}1$.

The reflector was used for the disappearance as the phenomenon was screened by trees from the refractor. Definition was too bad for a power of 200 to show a neat star-disc, and the brightness of Aldebaran seemed to obliterate the earth-lit limb near the point of contact, so that no projection could have been observed had it taken place.

At the reappearance the refractor was used, power 170. The star flashed up well within the bright limb, but as tree branches were partly veiling the O.G. this was somewhat diffuse owing to diffraction phenomena; there is no doubt, though, that very considerable projection occurred.

Both D. and R. were apparently instantaneous. As the star reappeared further within the limb than was anticipated, the recorded time of reappearance may be some fraction of a second too late.

Observations of the Transit of Mercury on 1924 *May* 7.

(*Communicated by the Secretaries.*)

The following observations were made at the Royal Observatory, Greenwich :—

Phase.	Observer.	Aperture.	Observed G.M.T.
		Inches.	h m s
Internal Contact, Egress	A.C.	3	May 7 17 36 6
,,	W.	6½	35 46
,,	P.M.	10	35 43
,,	F.J.	4	36 20
External Contact, Egress	A.C.	3	38 41
,,	W.	6½	38 53
,,	P.M.	10	38 26
,,	F.J.	4	38 11

The initials are those of Messrs. Crommelin, Witchell, Melotte, and Jeffries. The sky was clear but the sun's limb was unsteady and poorly defined. The altitude of the sun was about 10°. W. and P.M. both noted that through boiling at the sun's limb contact was made and broken again two or three times. The first formation of a drop between planet and limb was given by W. as 11^s before the time of contact. The best result to be drawn from the above figures is that internal contact took place at $17^h\ 35^m\ 45^s$.

At Cambridge the following observations were made with a 3½-inch Cooke Equatorial :—

	G.M.T.	Observer.
	h m s	
Egress, internal contact	17 35 47	W. M. Smart.
,, external ,,	17 38 14	H. E. Green.

Seeing : unsteady.

The following observations were made by Dr. W. H. Steavenson and Mr. R. L. Waterfield :—

Phase.	G.M.T.	Instrument.	Observer.
	h m s		
3rd contact	17 35 46·8	6-inch refractor	W. H. S.
3rd ,,	17 35 56·0	3-inch ,,	R. L. W.
4th ,,	17 38 41·4	6-inch ,,	W. H. S.
4th ,,	17 38 45·0	3-inch ,,	R. L. W.

The position of the observatory is :

W. long. $0^h\ 0^m\ 23^s{\cdot}10$.
N. lat. $51°\ 26'\ 9''{\cdot}9$.

Egress was observed under very poor conditions, the sky being filmy, with a slight drizzle falling. The limb of the planet was ill-defined,

and we estimate the probable error of our timing of the contacts to be between five and ten seconds.

Fr. Luis Rodés, S.J., of the Ebro Observatory, Tortosa, Spain, reports that he observed the egress under poor conditions with a 16 cm. equatorial of 210 cm. focal length and a magnification of 40, the internal contact taking place at $17^h\ 35^m\ 29^s$ G.M.T.

The following observations were made by Professor S. D. Tscherny (T) and A. A. Karamischev (K) at Kiew :—

1. *Instruments.*—Professor Tscherny : Merz-Repsold refractor, aperture 243 mm., stopped down to 125 mm., magnifying power 145.

Karamischev : Goerz refractor, aperture 70 mm., magnifying power 58.

2. *Observations of Contacts.*

	h m s	Obs.	Remarks.
III. Contact, Greenw. Mean Time.	17 35 47·2	T	Good.
	17 35 59·1	K	Windy.
Bisection of Mercury, Greenw. Mean Time.	17 37 13·9	T	Satisfactory.
	17 37 26·9	K	Windy.
IV. Contact, Greenw. Mean Time.	17 38 31·7	T	The limb of the Sun was boiling.
	17 38 52·6	K	The limb was boiling.

3. *Results of Kiew Observations of Contacts.*

	Obs.	May 1924.	N.A.	O−C.	C. de T.	O−C.	B.J.	O−C.
		d h m s	m s	s	m s	s	m s	s
III. . .	2	7 17 35 53·2	36 8·6	−15·4	36 8·7	−15·5	36 15·0	−21·8
Bisection .	2	7 17 37 20·4	37 38·3	−17·9	37 38·5	−18·1	37 44·8	−24·4
IV. . .	2	7 17 38 42·2	39 7·9	−25·7	39 8·3	−26·1	39 14·6	−32·4

4. *Observations with the Filar Micrometer.*—The atmosphere was in agitation, and therefore the observations were difficult. The diameter of Mercury was measured eight times, with the following results :—

	r
	0·391
	0·410
	0·347
	0·375
	0·359
	0·339
	0·349
	0·345
Mean	0·3644

The value of one revolution of the micrometer screw for the temperature at the time of observation (+11°·5 R) is 28″·3465, and therefore we have for the diameter of Mercury 10″·33, with the probable

error $\pm 0''\cdot170$. The diameter of Mercury given in the *Nautical Almanac* is $11''\cdot98$, and therefore $O - C = -1''\cdot65$.

At the Nizamiah Observatory the transit was observed under fairly good atmospheric conditions by Mr. T. P. Bhaskaran. Of the four contacts only the last two were visible, the ingress of the planet happening about $2\frac{1}{2}$ hours before sunrise at Hyderabad. The morning was cloudless, but at times a slight haze prevailed, which however was never dense enough to obscure the planet. The telescope employed was the 15-inch Grubb Equatorial (aperture stopped down to 6 inches), and a sun-glass was screwed on to the eyepiece (power 150). The time was taken at the telescope with a marine chronometer, whose error was determined directly from Transit Observations on the night of May 8 and the preceding night. The following are the times of the two contacts that were observed:—

Egress.	G.M.T. h m s
Black drop began to form	17 35 41
Geometrical Contact (Internal)	17 35 56
External Contact	17 38 16 (probably too early)

The definition at the sun's limb became poor at the time of the contacts and the image began to get unsteady.

During the transit the planet was observed with the telescope at frequent intervals. The disc was quite black and round, the edge was sharp and well defined, and no luminous ring or halo could be seen round the planet. A small white spot was at first occasionally visible about the centre of the disc, which, later on, became more persistent and just before the time of the contacts it was fairly conspicuous.

The co-ordinates of the observatory are:

Lat. $17^\circ\ 25'\ 54''\cdot3$ N.
Long. $5^h\ 3^m\ 48^s\cdot98$ E.